FINITE ELEMENT ANALYSIS

using

BALANCED LOADING

with

MINIMAL SUPPORTS

Model Types Demonstrated

- Axisymmetric
- 2D Plane Stress/Strain
- 3D Solid

- 2D model with one line of symmetry
- 2D model with two lines of symmetry

- 3D model with one plane of symmetry
- 3D model with two planes of symmetry
- 3D model with three planes of symmetry

- 3D model with cyclic symmetry subject to tension and internal/external pressures
- 3D model with cyclic symmetry subject to tension and/or torsion

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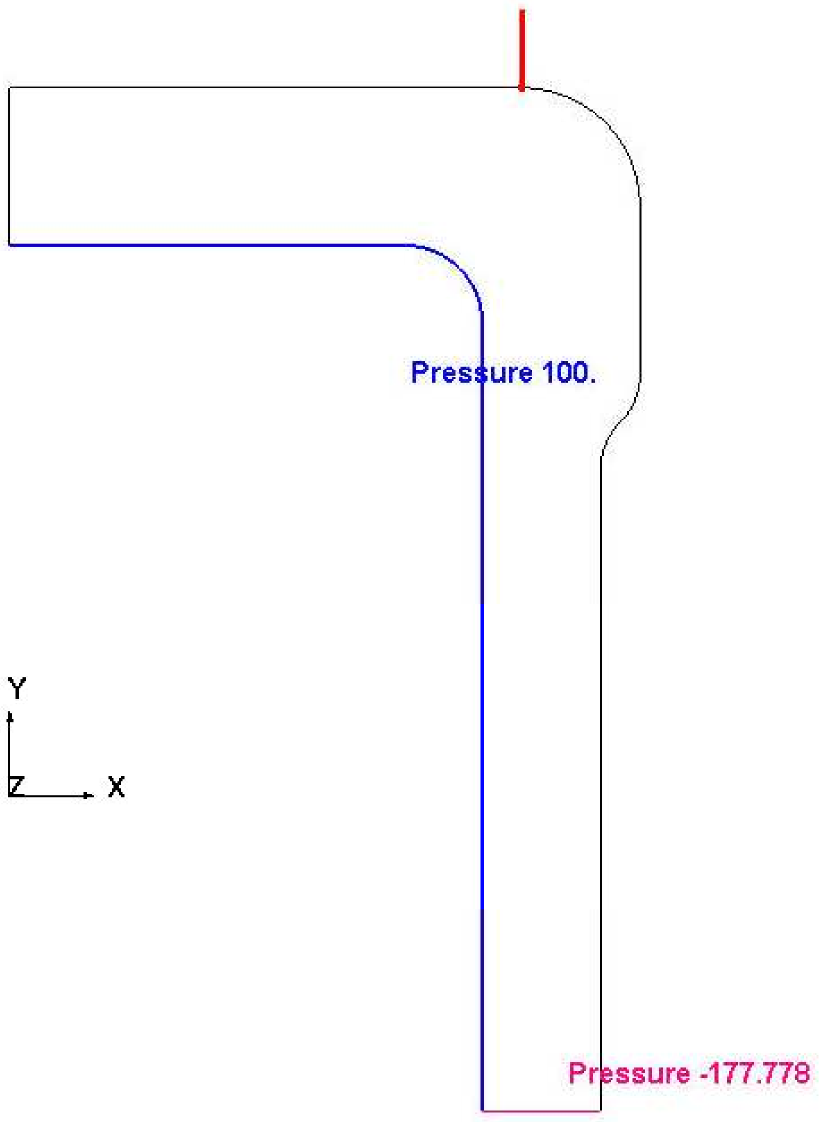
Axisymmetric Models

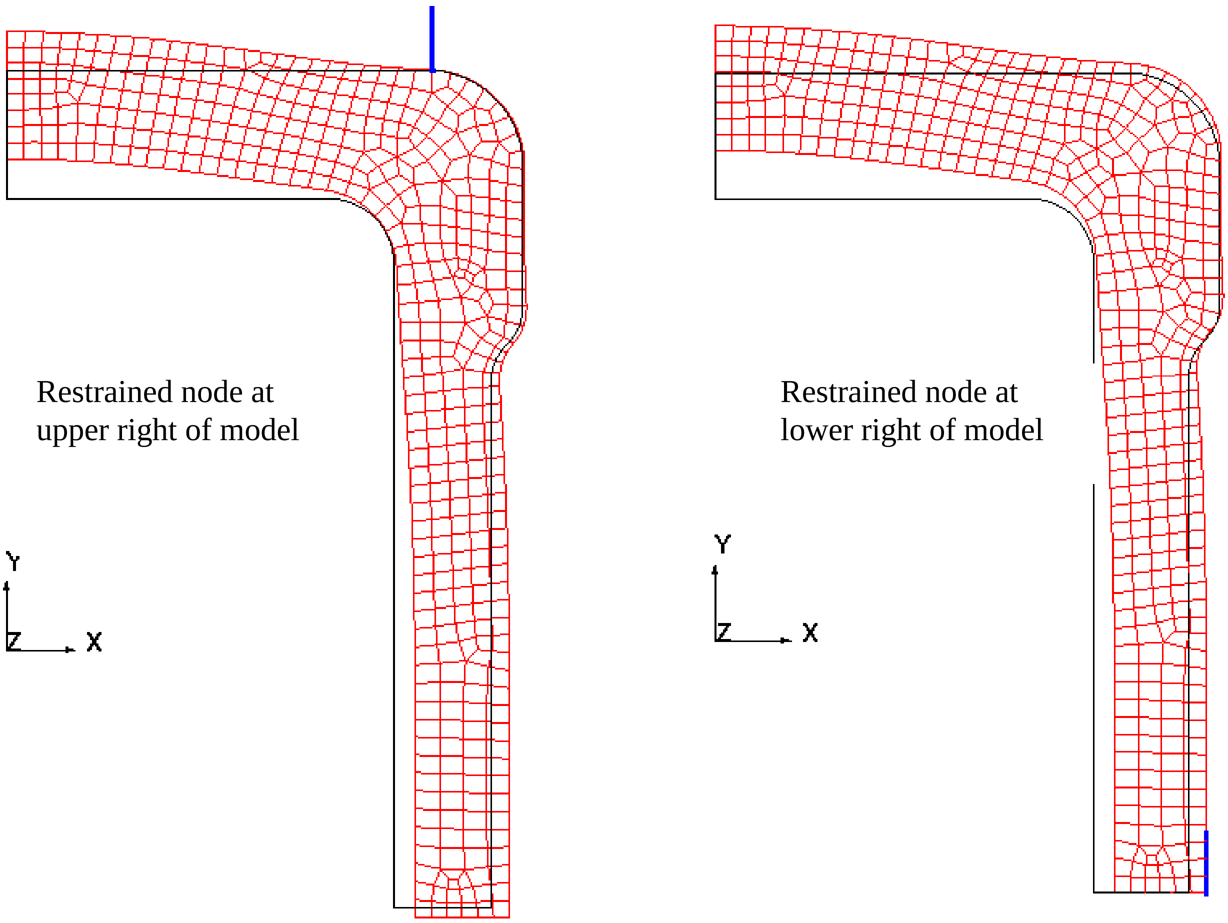
Axisymmetric models are used to analyse circular objects with constant geometry section and constant boundary conditions through 360 degrees. This is a particular useful method for analysing pressure vessels where the 3D geometry of the part is accurately represented by a 2D mesh. The model is created in a 2D plane, most solvers use the X global axis for radial position and the Y global axis for position along the axisymmetric axis of the part.

The model has only one unrestrained rigid body freedom, that is along the axisymmetric axis (or Y axis in Roshaz and most other systems) that requires fixing for a solution to be possible in FEA

In the following example a uniform internal pressure of 100 has been applied to the inner surface of a pressure vessel end cap. The thrust in the Y direction that this pressure creates has been balanced by a pressure applied to the vessel wall through thickness where the model has been terminated. The net force applied to the model in the Y axis is then zero. A single node has been fixed in the Y direction. This restraint is sufficient to allow the solver to run. The restrained node does not react any load to ground (other than negligible round off errors) and does not affect the stress contour pattern in the model. The selection of the restrained node is arbitrary. The deformed shape of the mesh remains the same whatever node is chosen to "ground" the model, only the position of the datum with zero Y deflection is different.

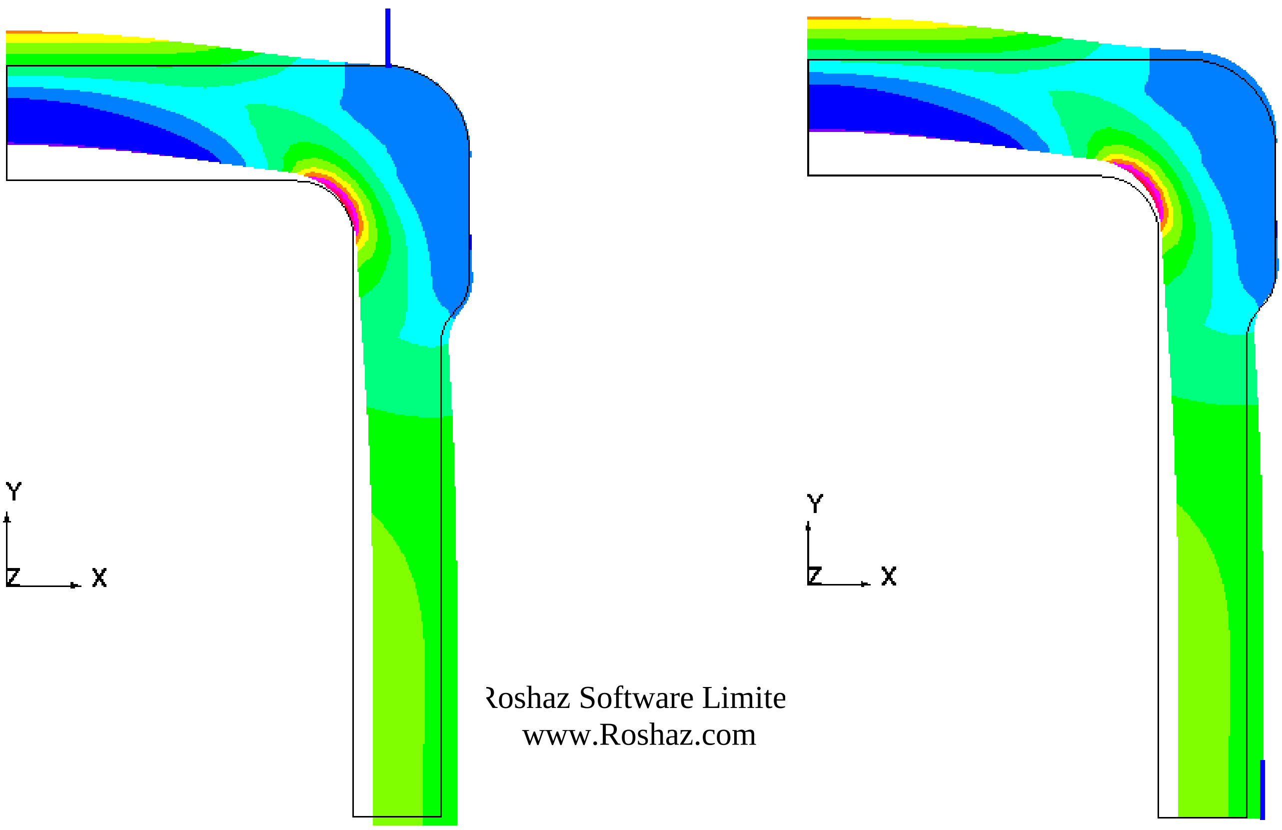


Exaggerated Mesh Deformation



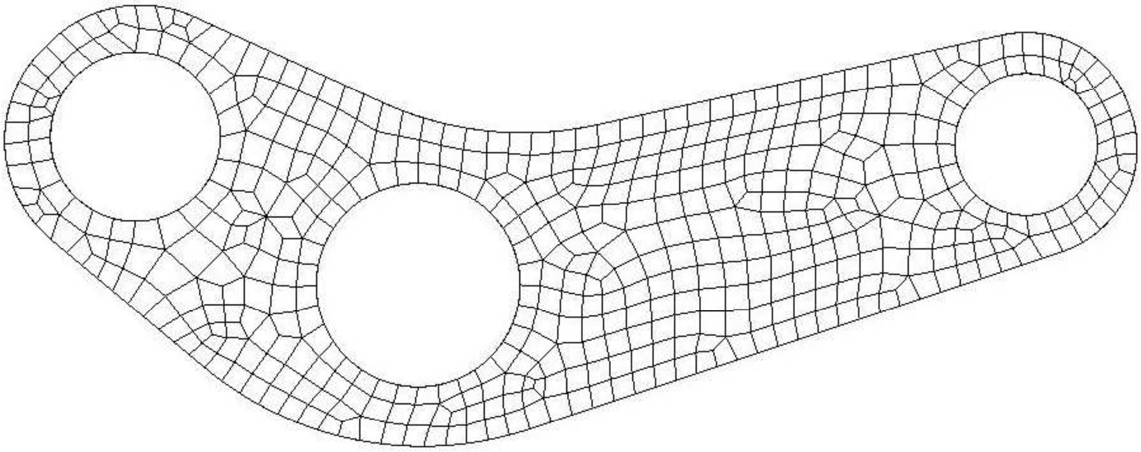
The deformed shape in both of the above plots is identical. Internal strains and hence stresses are the same and are not affected by the node selected for the restraint.

Maximum Principal Stress

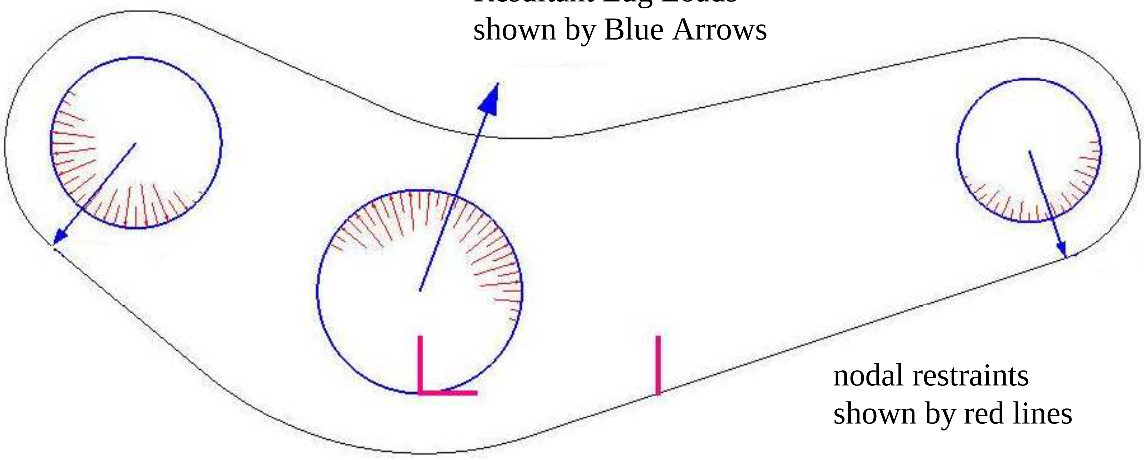


2D Plane Stress/Strain

The mesh below has been used to model a simple crank lever subject to a statically determinate three point loading system.



The boundary conditions used in this model are shown below.

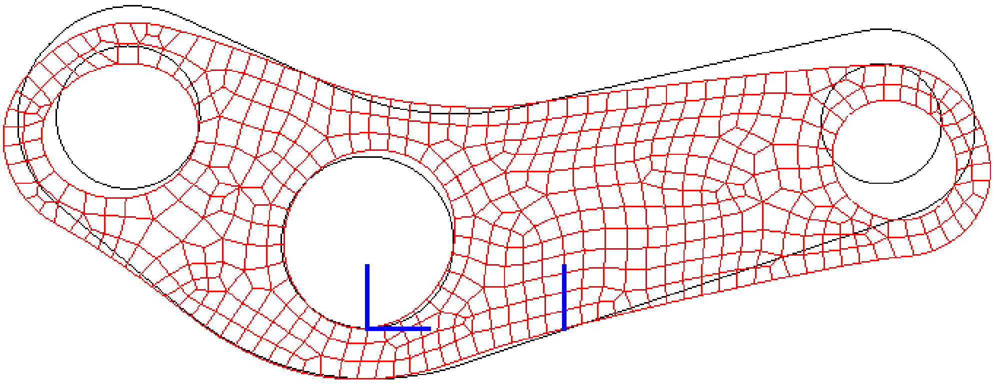


The lug loads have been applied using a Gencoz pressure distribution. The sum force of the three applied loads to each lug is zero. The model is also in moment equilibrium.

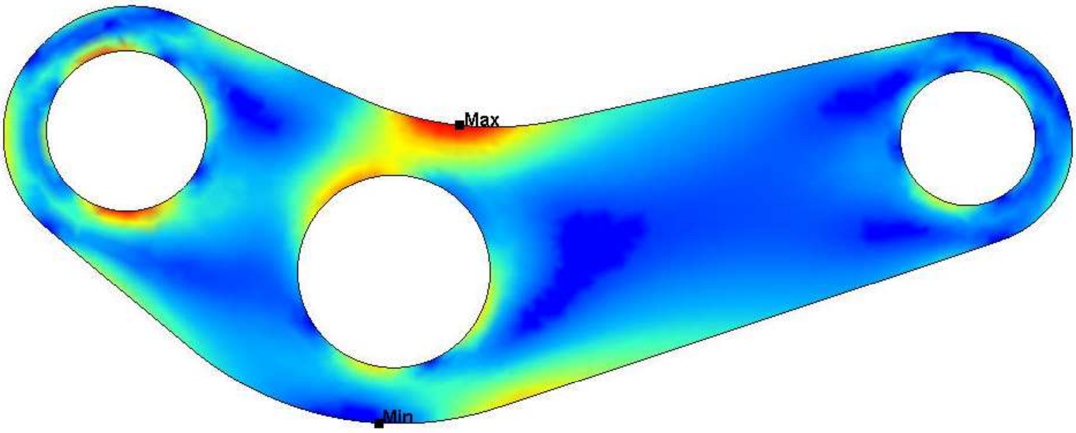
In 2D an object has three rigid body modes of movement, two in translation and one rotation. Without restraints the part is free to move up/down, right/left and spin. The minimal supports required to prevent all rigid body motions are applied using the so-called "2 - 1" method. One point is selected to apply two translational supports. All movement apart from rotation is then restricted at this point. With just this point restrained the part is still free to spin about this point. Rotation of the part is then stopped by applying a single restraint at a second point in a direction normal to a line drawn from the first point to the second point. A total of three restraints has been applied, this equals the number of rigid body freedoms for a minimal support condition. Any fewer supports and the model is under restrained and no static solution is possible, any more than three supports and the model is over restrained and is no longer a statically determinate problem.

The points chosen for the supports should be well separated. A one hundred percent perfect load and moment balance is never possible, there will always be a negligible round off error present. Therefore if the two points are very close together, the effect will be that the round off errors get magnified by the small moment arm. Lengthening the distance between the points and hence increasing the available moment arm minimises support reactions caused by round off errors.

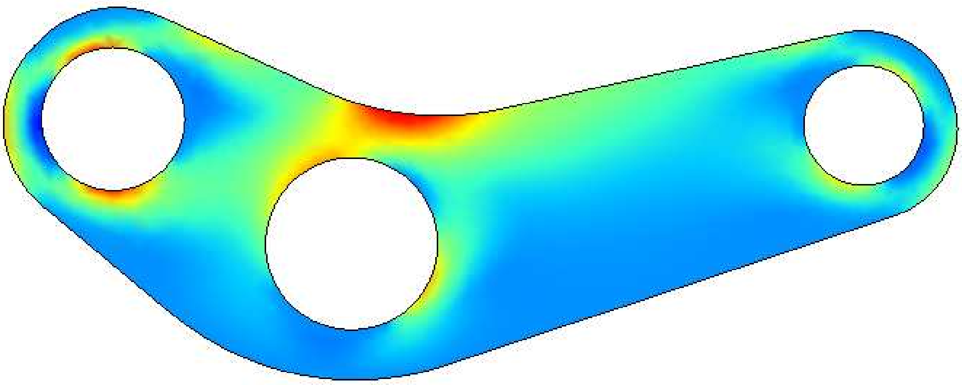
Exaggerated Mesh Deformation



Von Mises Stress contours



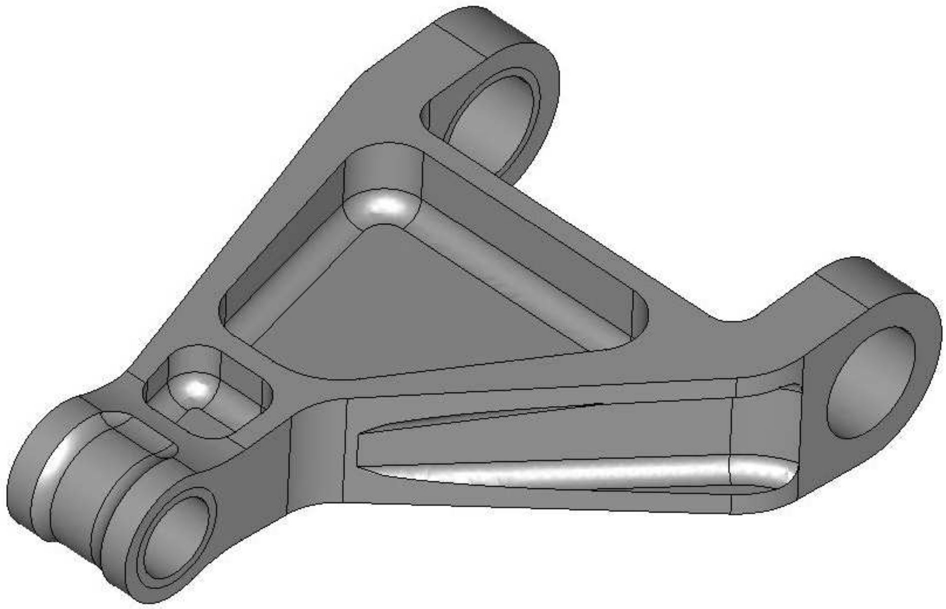
Maximum Principal Stress



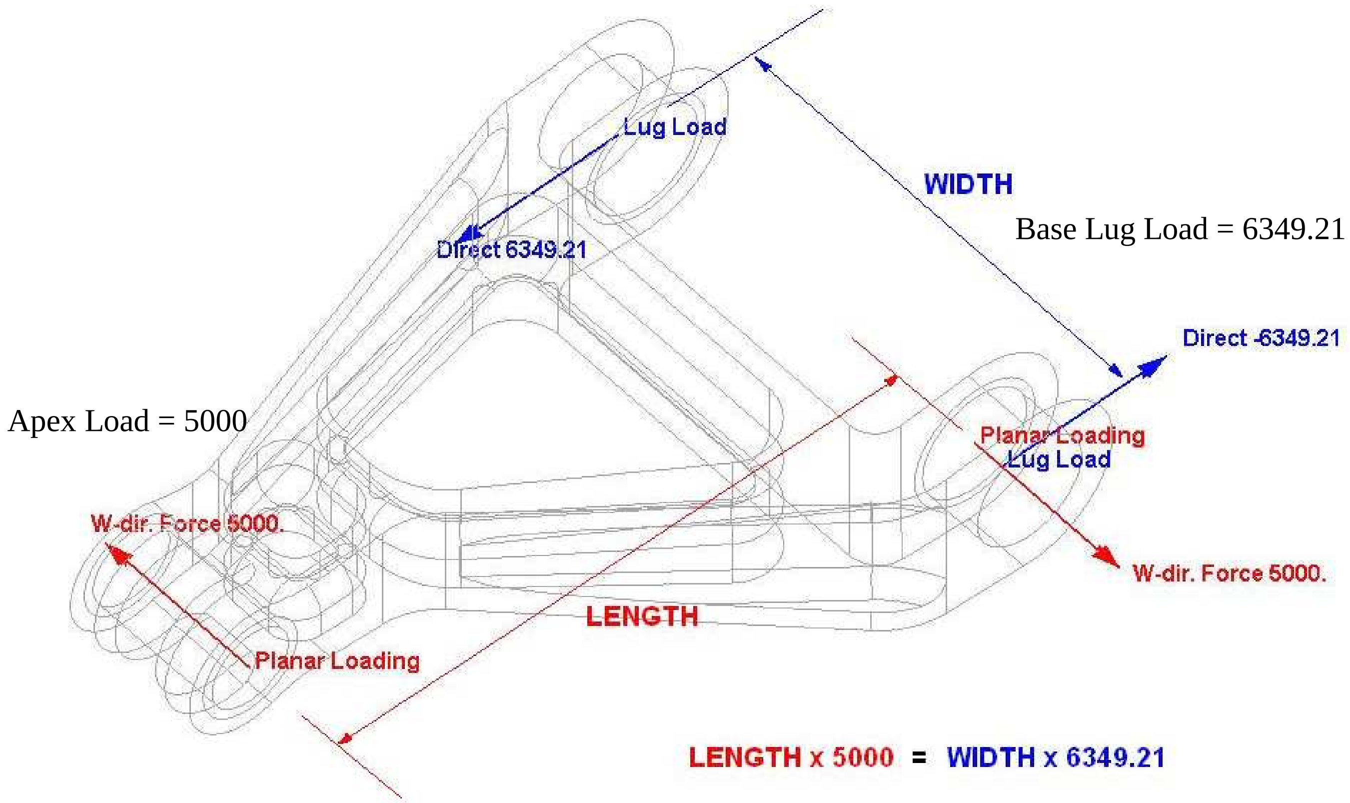
As can be seen the supported nodes have not interfered with the stress contour result plots.

3D Solid Model

The model here is a shock absorber torque link. As the name implies the purpose of this component is to transmit any axial moment applied to a piston on to the main cylinder of the shock absorber. There are a pair of torque links acting together which articulate as the shock absorber strokes in and out. Torque is transmitted via a pair of equal and opposite forces acting together to create a moment. One force is transmitted as a shear load up the piston and is reacted by the bearings with the cylinder whilst the other load is a shear passing through the torque link (the so called apex load).

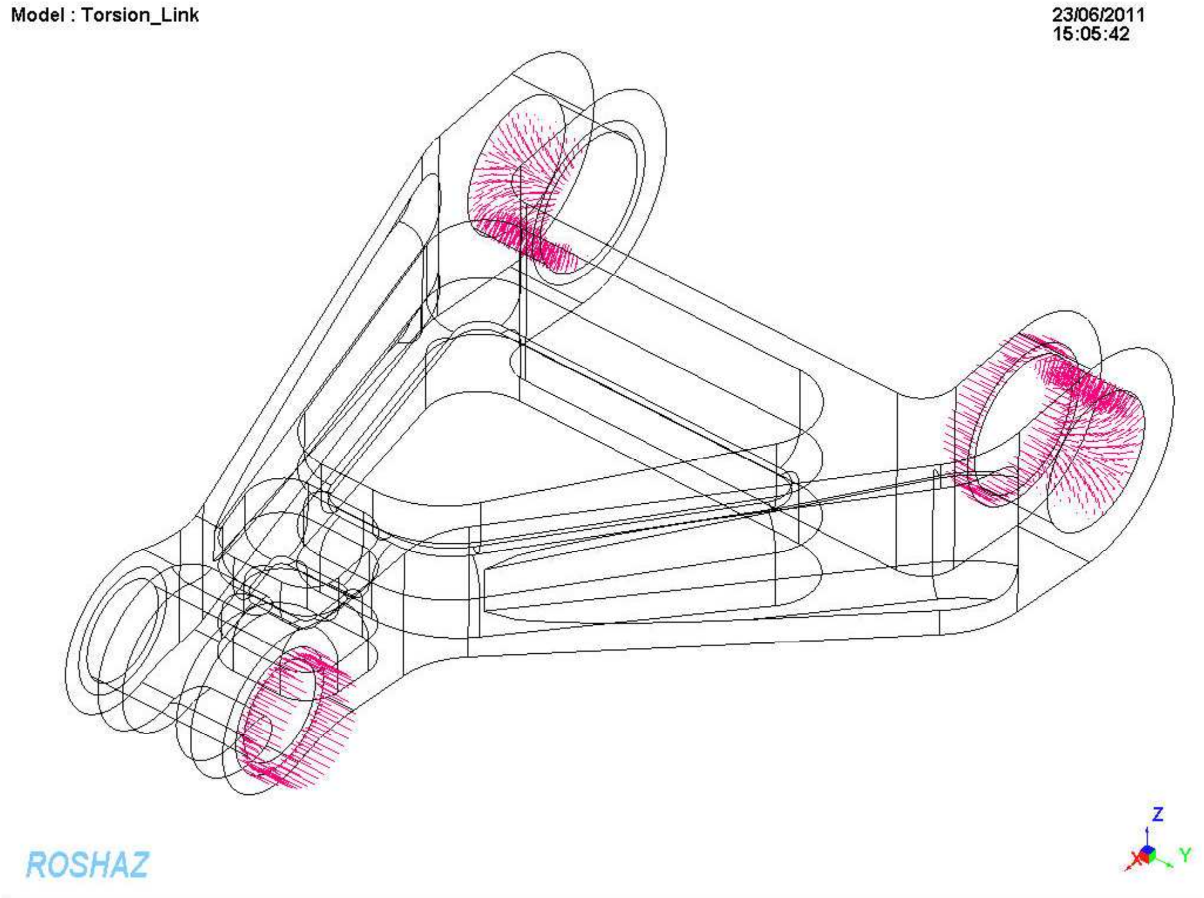


Free body diagram of loading on a torque link

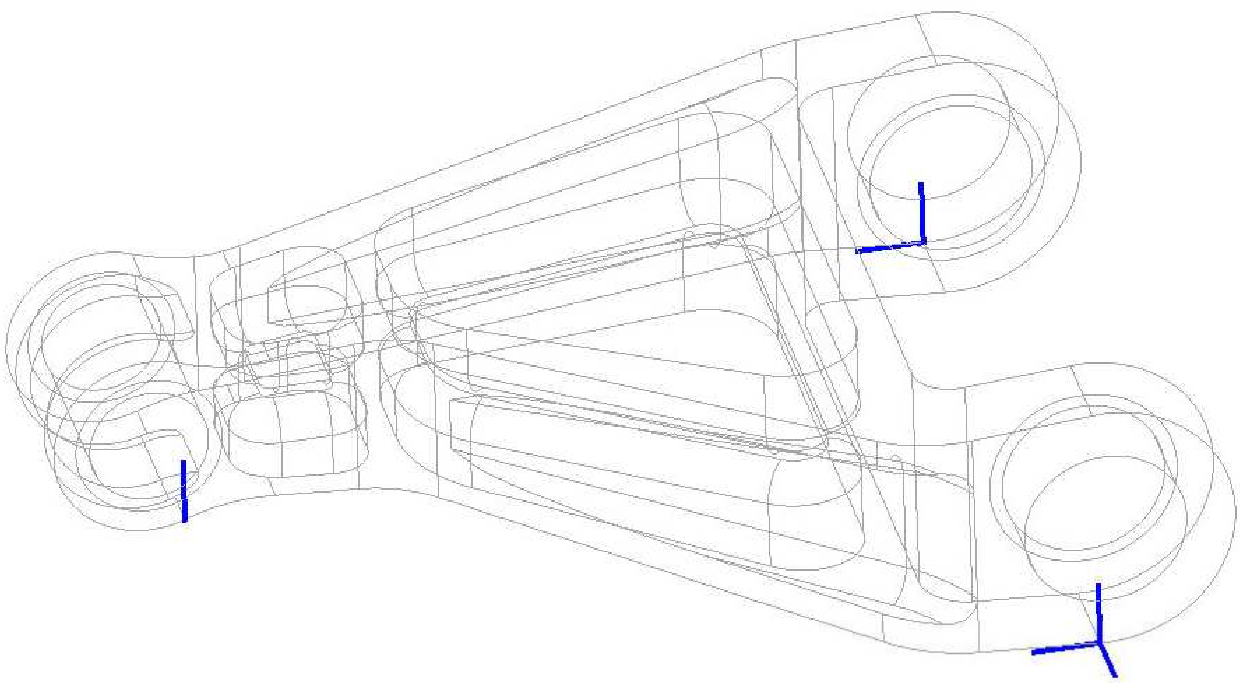


The moment created by the equal and opposite loading on the side walls of the apex and base lugs is reacted by push and pull loads in the base lugs. The loading as shown is in both force and moment balance.

Pressure loading applied to the torque link



Minimal 3 - 2 - 1 supports used to "ground" the model

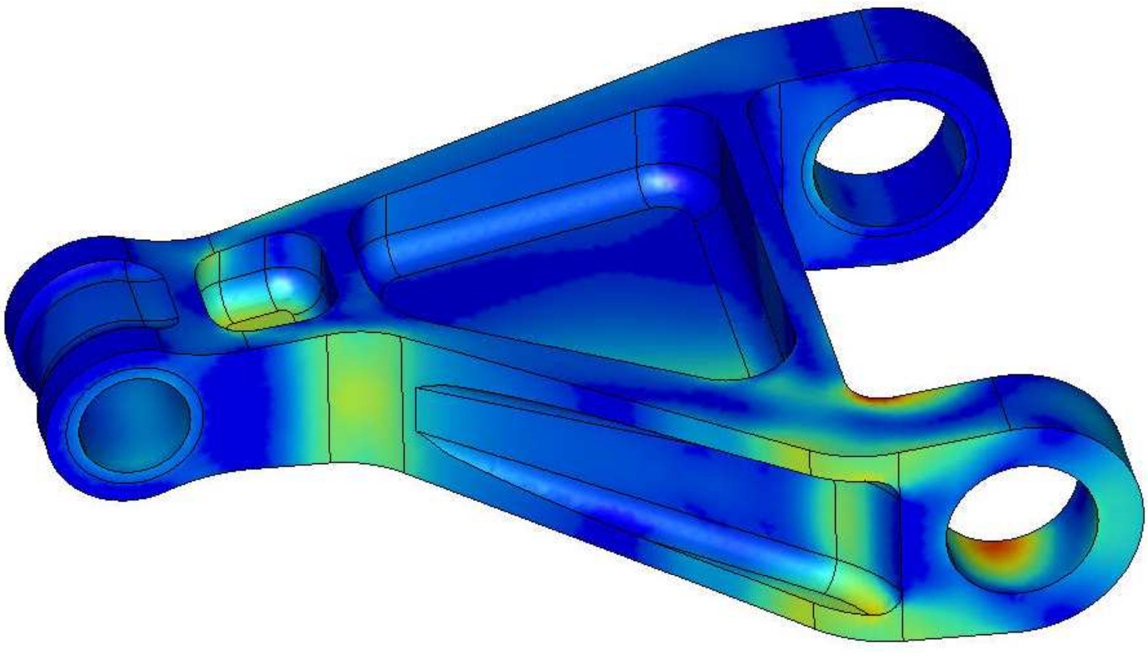


A 3D object has six rigid body freedoms, three in translation and three in rotation. Therefore at least six supports are required to remove these six freedoms before a static analysis is possible. With the so called 3-2-1 approach three points are selected. As with the 2D model, these points should be reasonably well separated. The first point is fixed in all translational directions. With this point alone, all the three translation rigid body motions have been taken out, the object is still free to rotate about this point in all three axes. A second point is then selected and fixed in the two directions normal to a line joining the first and second points. By doing this the object is not restricted in expansion or contraction (with a thermal input) by these points. Adding this second point with it's two supports takes out two of the three rigid body rotations. The object is now only free to rotate about an axis defined by a line joining the first and second points. This remaining rotational freedom is removed by fixing a third point in a direction normal to the plane defined by the three points. Of course this third point must not be in line with the first two points! The 3-2-1 method can use any convenient axis system. If the object has no easily identifiable global planes available then a local axis system will be used instead.

A good test of the supports chosen is a thermal analysis. The part should be freely capable to thermally expand or contract with zero internal strains and stresses.

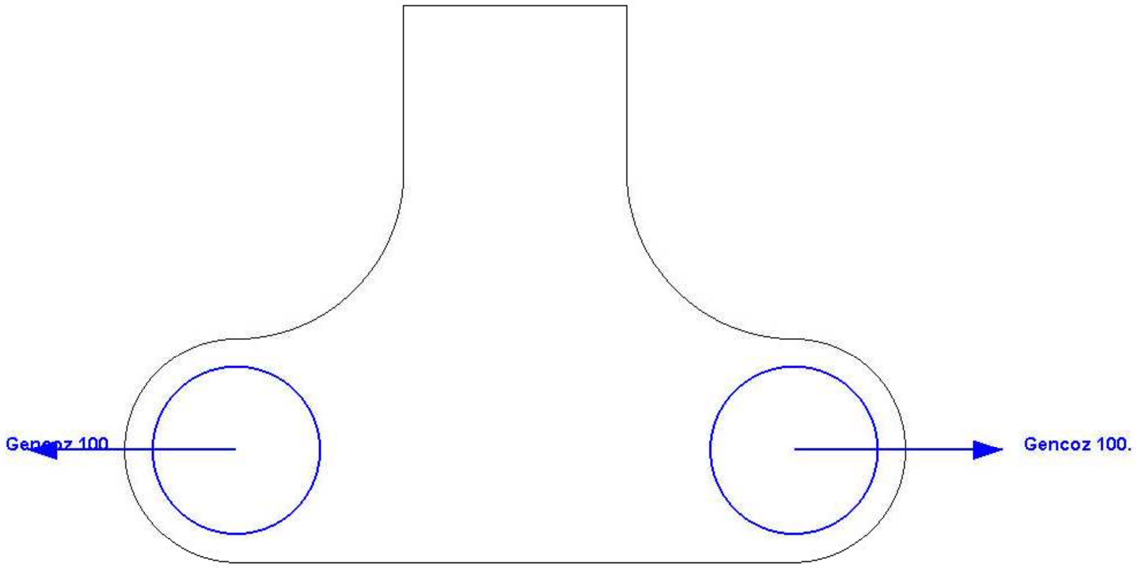
As with the 2D example with a correctly applied balanced load system the 3-2-1 minimal supports will only react negligible round off errors and will thus not interfere with contour stress plots.

Von Mises Stress



2D Model with one line of Symmetry

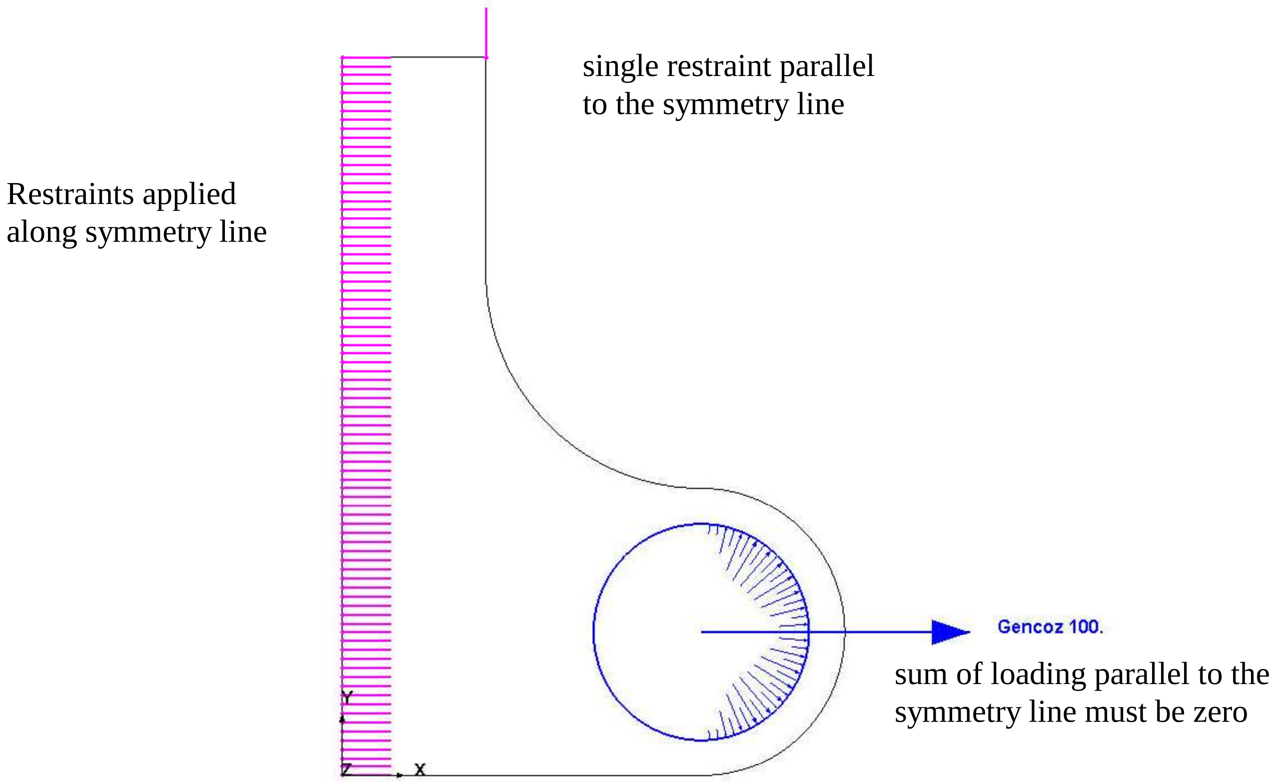
The following model can be split vertically down the centre to form a mirrored pair. This is the line of symmetry.



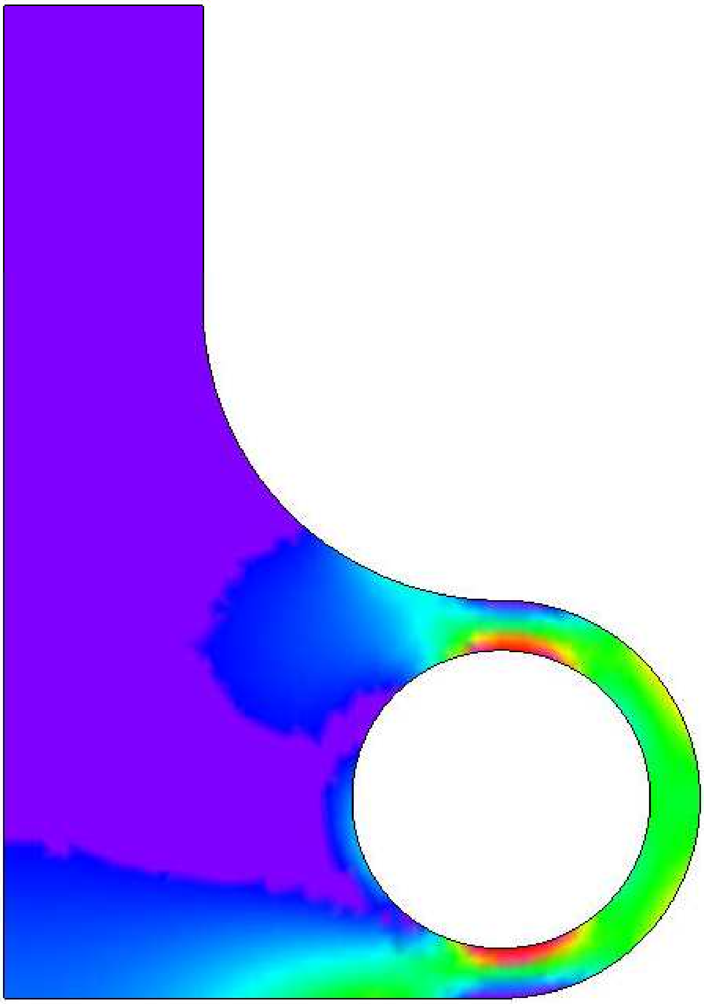
In a symmetry model restraints normal to the line of symmetry are placed along the line. As well as preventing rigid body motion normal to the symmetry line the model is also prevented from being able to spin.

With this in place the model has just one rigid body freedom remaining, that is sliding along the symmetry line. Therefore one support parallel to the symmetry line is required to eliminate this last rigid body freedom.

In the above model for a balanced load condition the sum of forces in the Y direction must be zero.

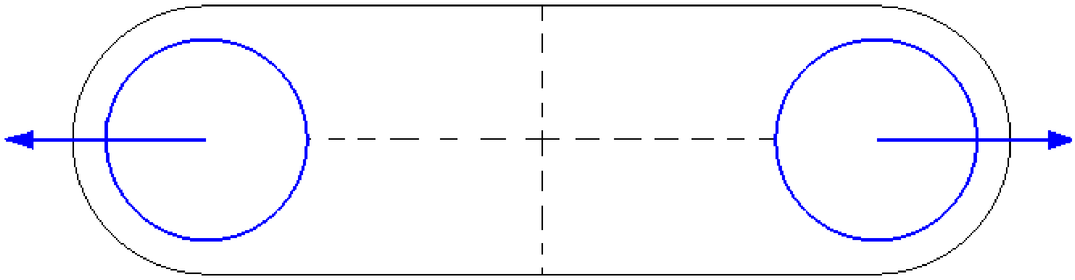


Von Mises stress contours

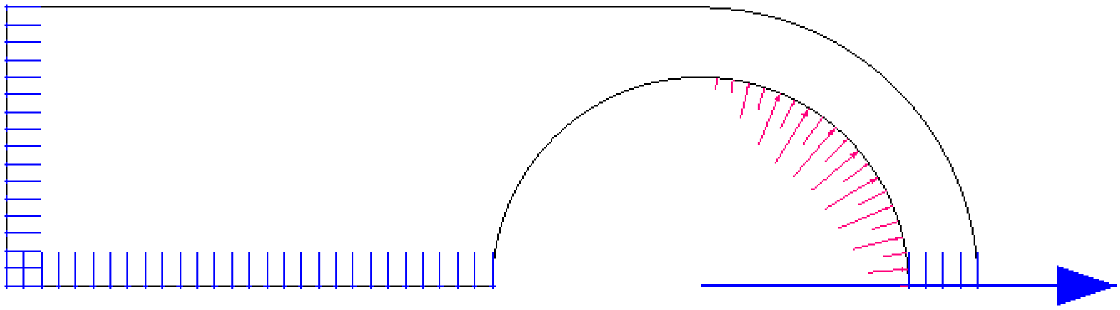


2D Model with two lines of Symmetry

The following model can be split vertically and horizontally along the dashed lines



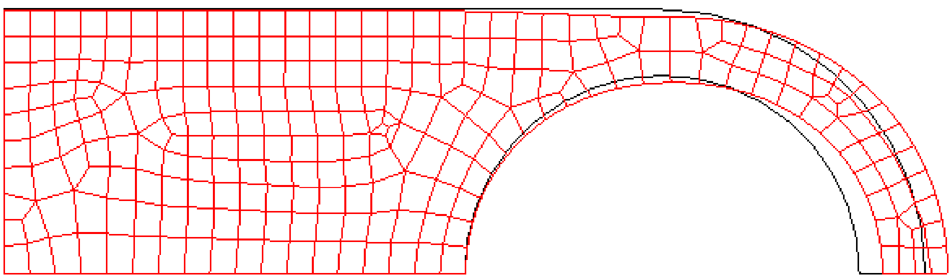
This reduces the model analysed to a quarter of the original

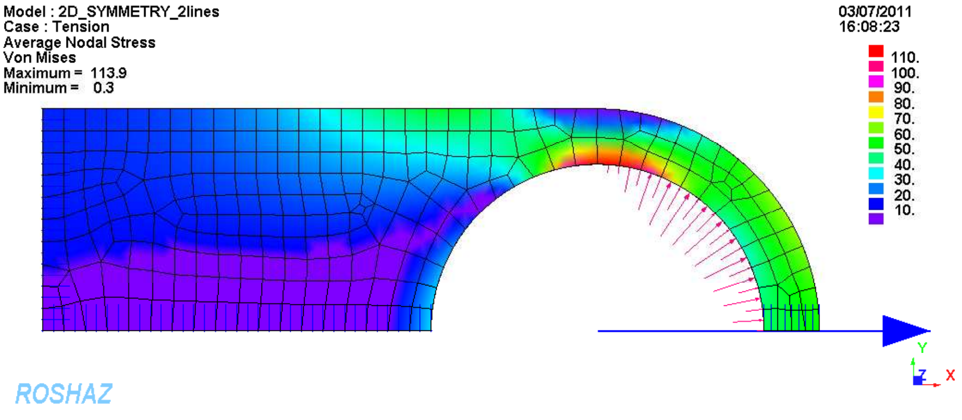


Restraints have been applied normal to each line of symmetry along the lines of symmetry.

With these supports applied the model has no rigid body freedoms, no additional supports are required.

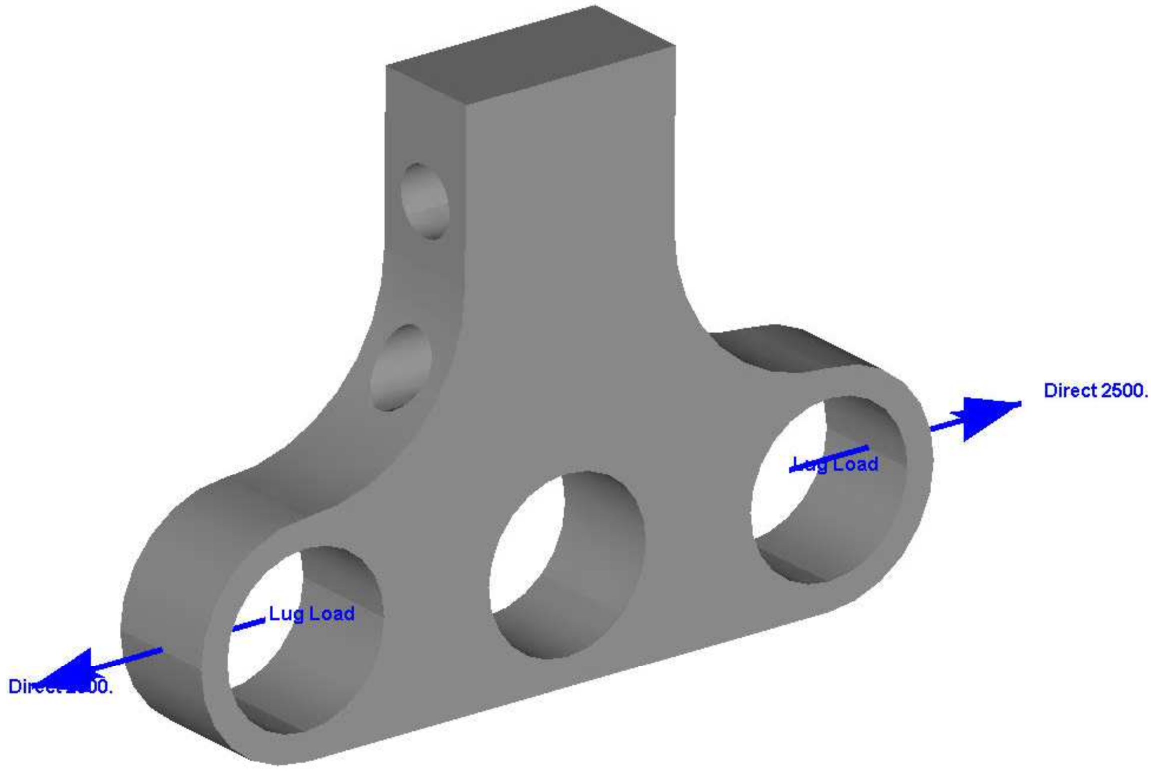
Exaggerated Deformation





3D Model with one plane of Symmetry

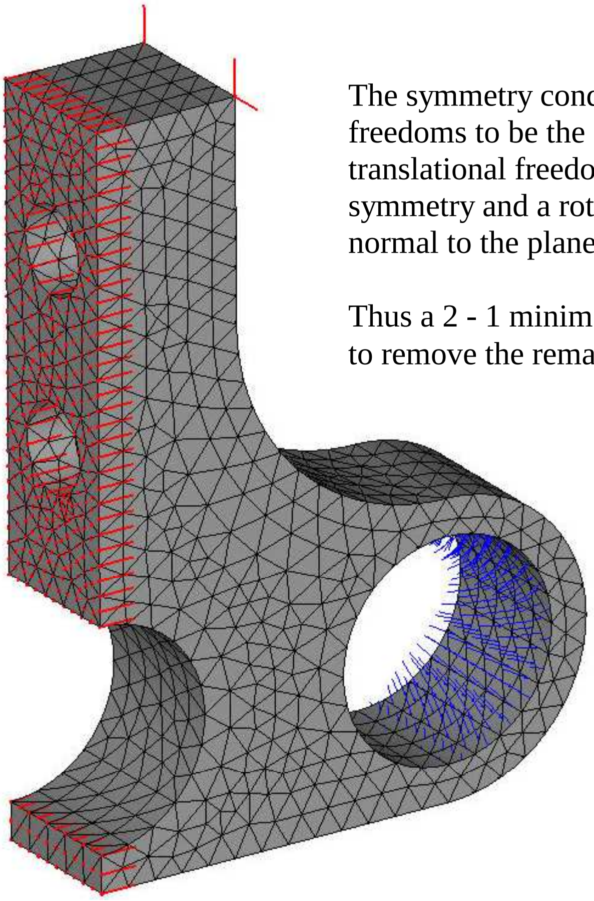
The following model can be split vertically down the centre to form a mirrored pair. This is the plane of symmetry.



The following plot shows the boundary conditions applied to the symmetric half model.

The symmetry condition reduces the rigid body freedoms to be the same as a 2D model with two translational freedoms parallel to the plane of symmetry and a rotational freedom about an axis normal to the plane of symmetry.

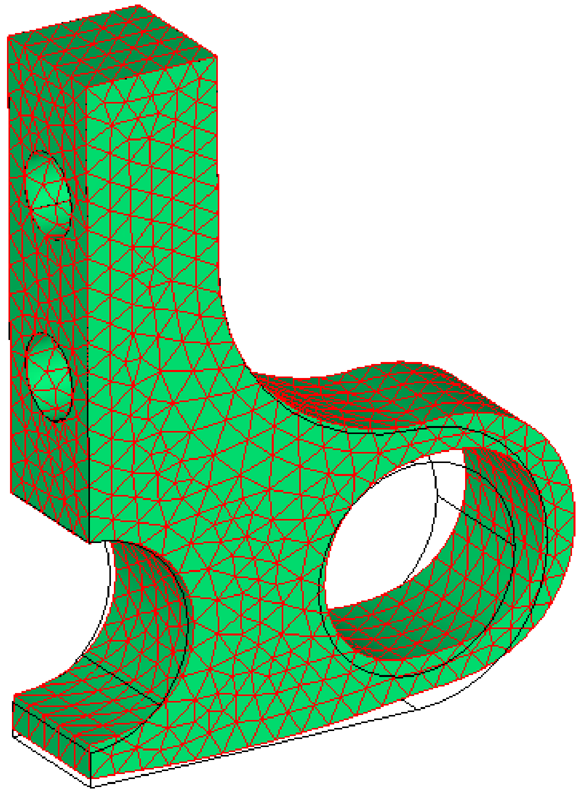
Thus a 2 - 1 minimal support approach can be used to remove the remaining rigid body motions.



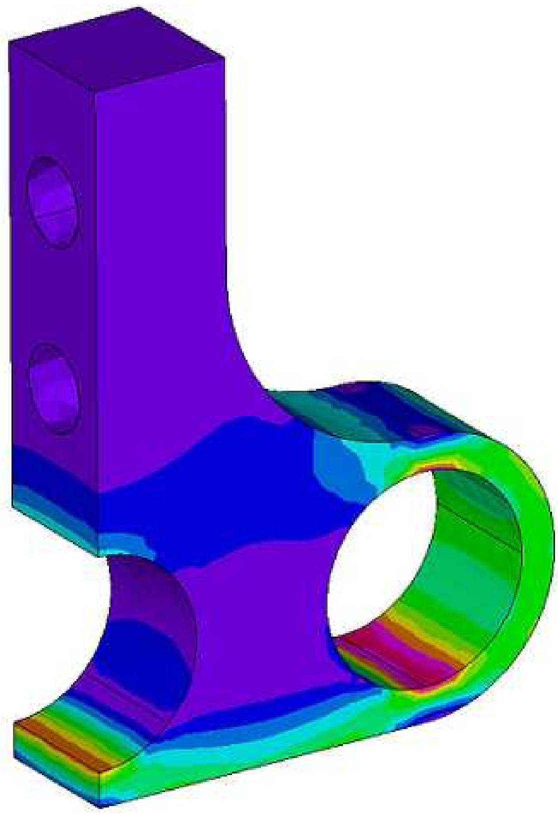
Restraints normal to the plane of symmetry have been applied.

This removes the two out of plane rotational rigid body freedoms and translation normal to the symmetry plane

Exaggerated Deformation

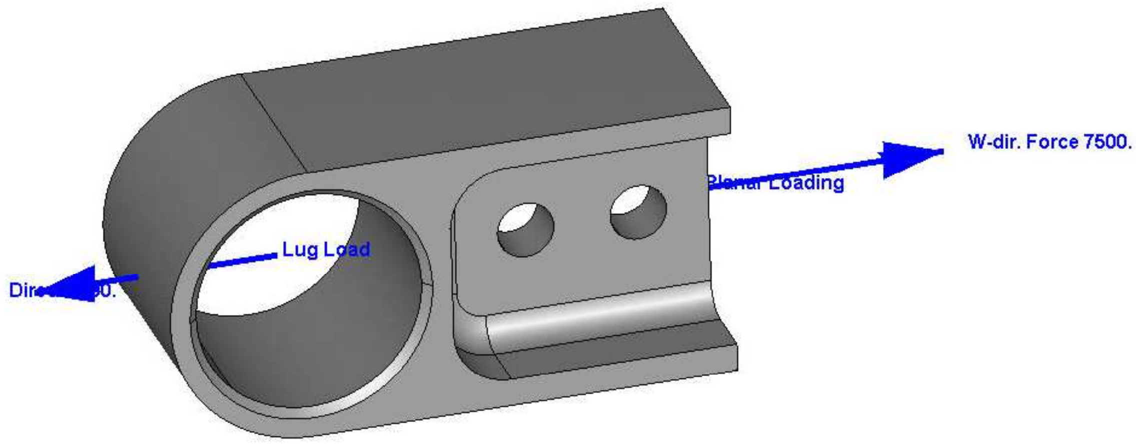


Von Mises stress contour plot

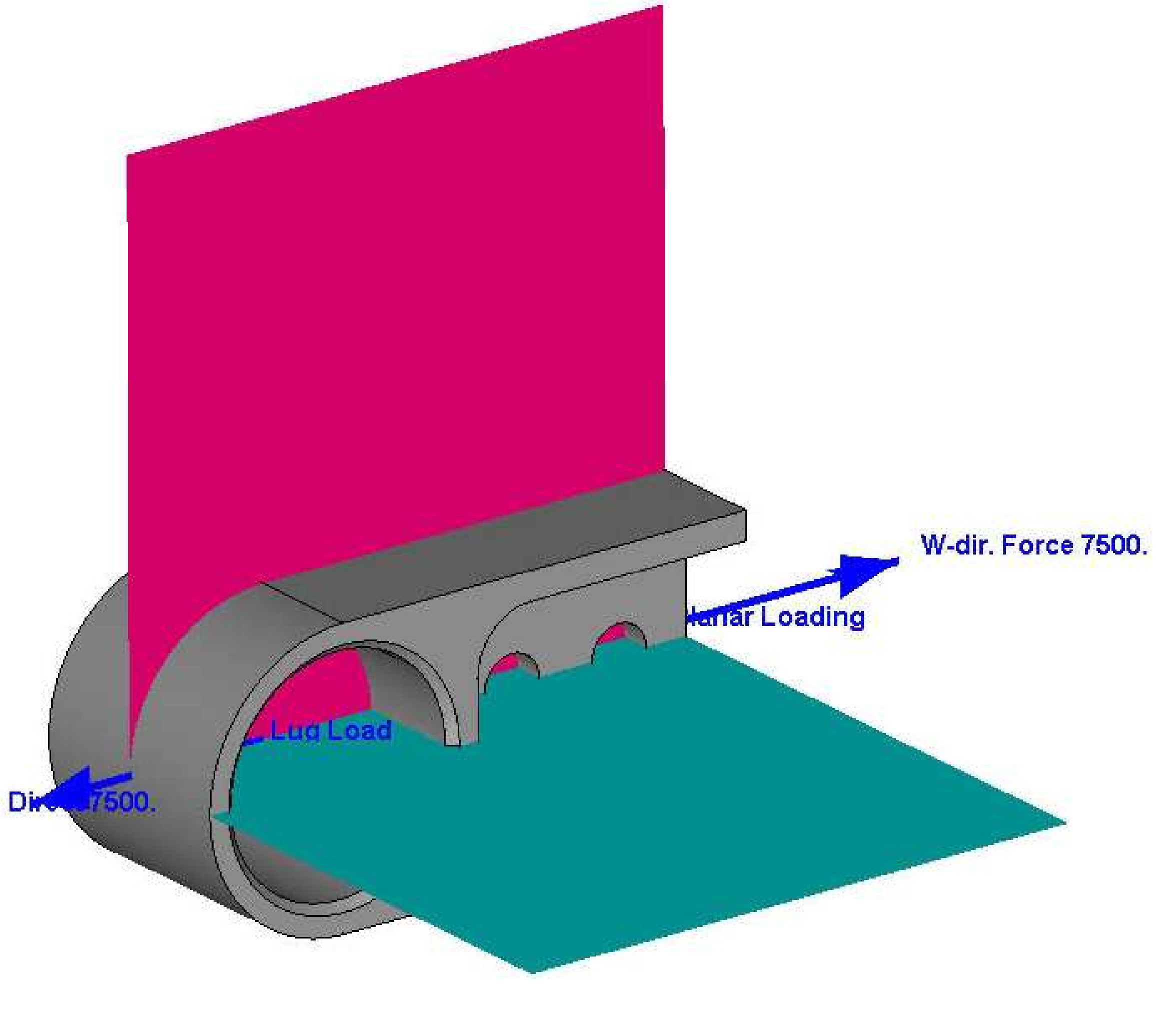


3D Model with two planes of Symmetry

The following model can be split vertically and horizontally. This produces a quarter model of the original.

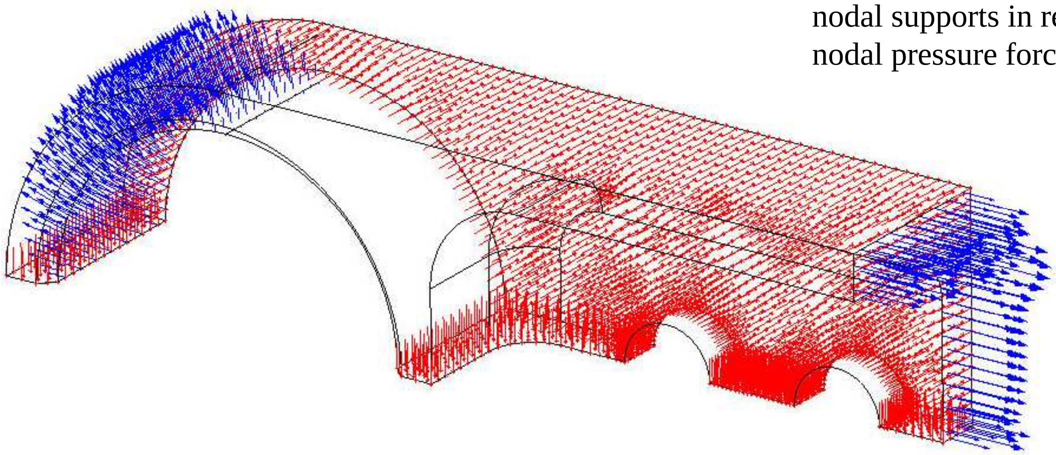


Planes of symmetry indicated in magenta and cyan.

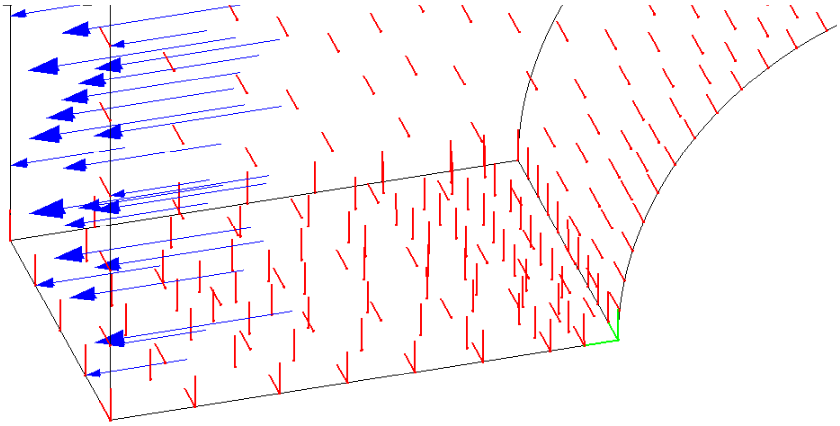


Boundary conditions applied to the quarter model

nodal supports in red
nodal pressure forces in blue

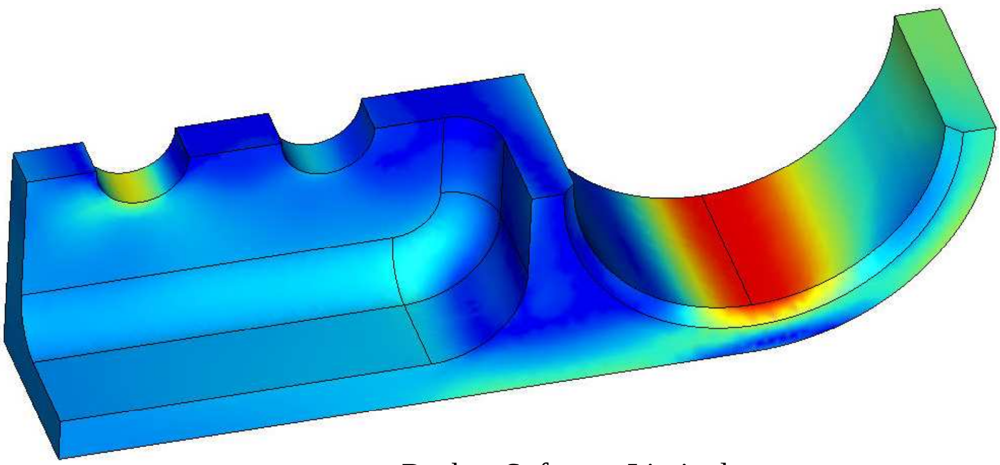


After applying the two symmetry support conditions the model has one rigid body freedom remaining. It can slide length wise along the line mutually parallel to both symmetry planes. Therefore a single additional support in this direction is required.



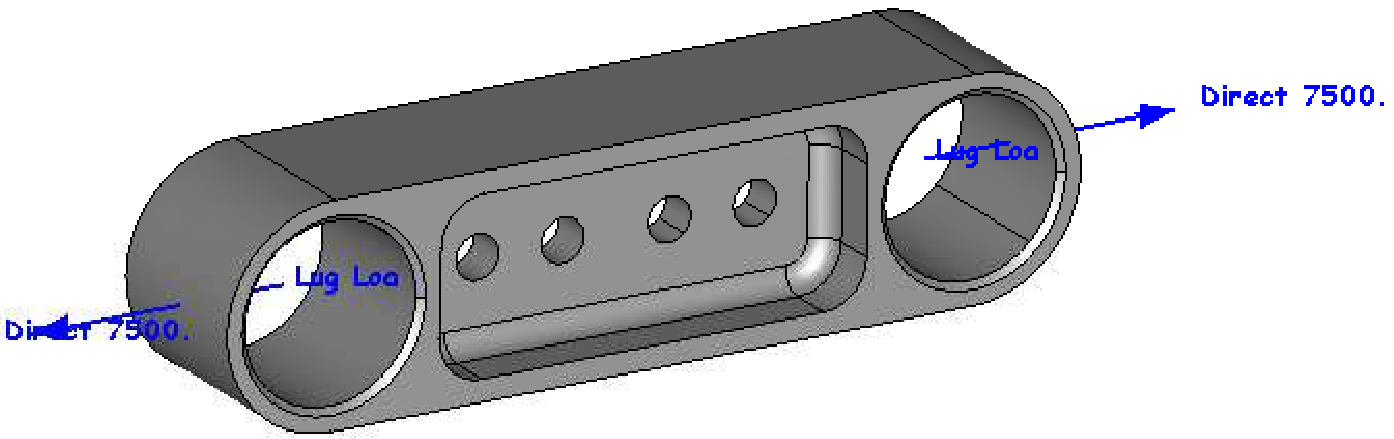
Along the intersection of the symmetry planes nodes have two support conditions applied. A single node on this intersection line has been selected to have the additional support applied, shown here in green. This single node is then supported in all three translational directions.

Von Mises stress contour plot

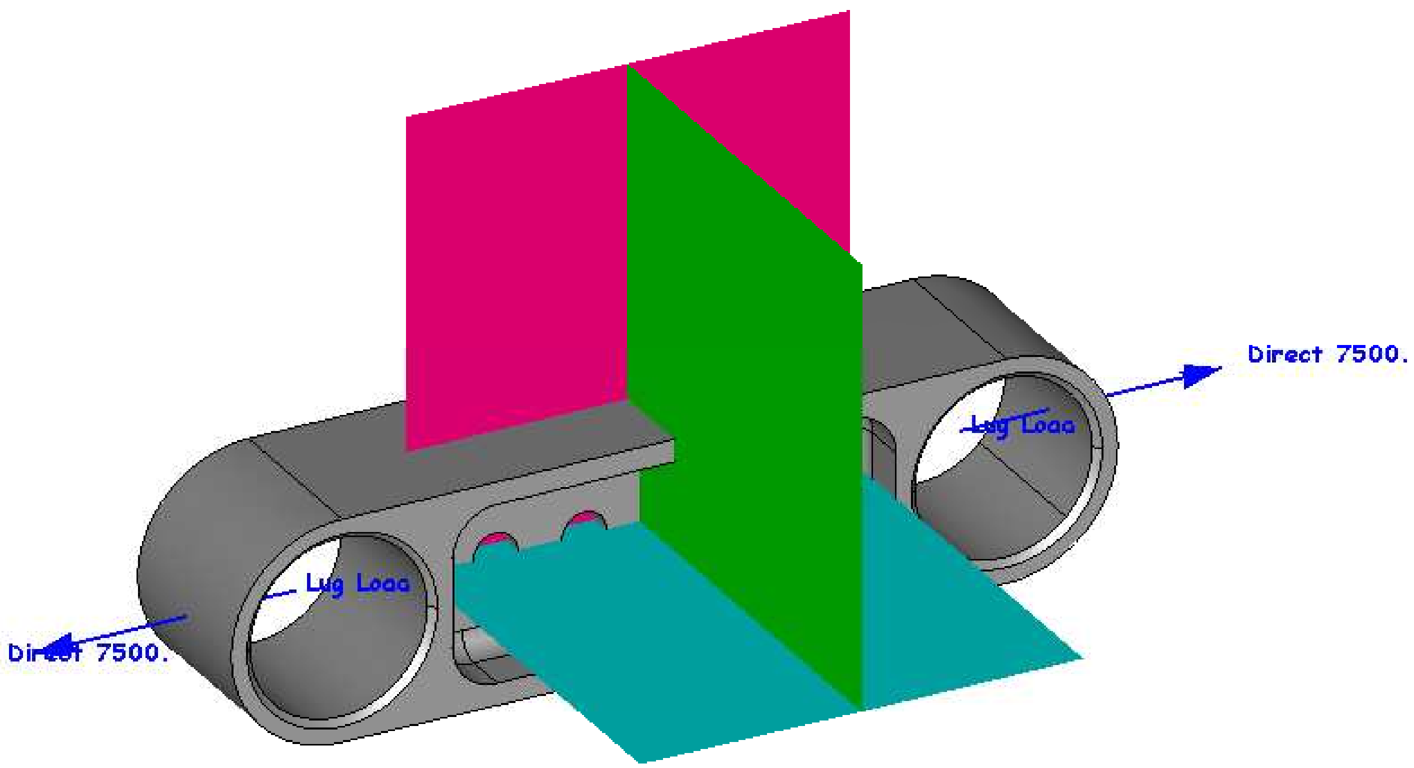


3D Model with three planes of Symmetry

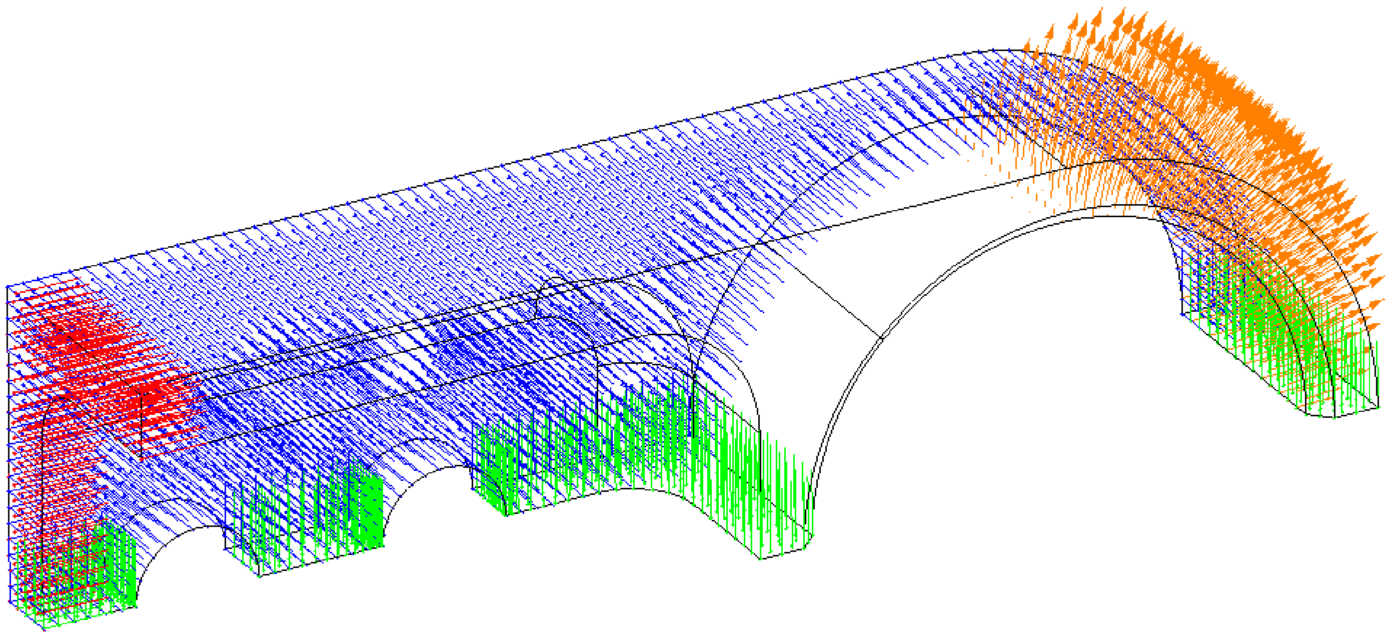
The following model can be split vertically, horizontally and at a mid way plane. This produces a one eighth model of the original.



Planes of symmetry indicated in magenta, cyan and green.



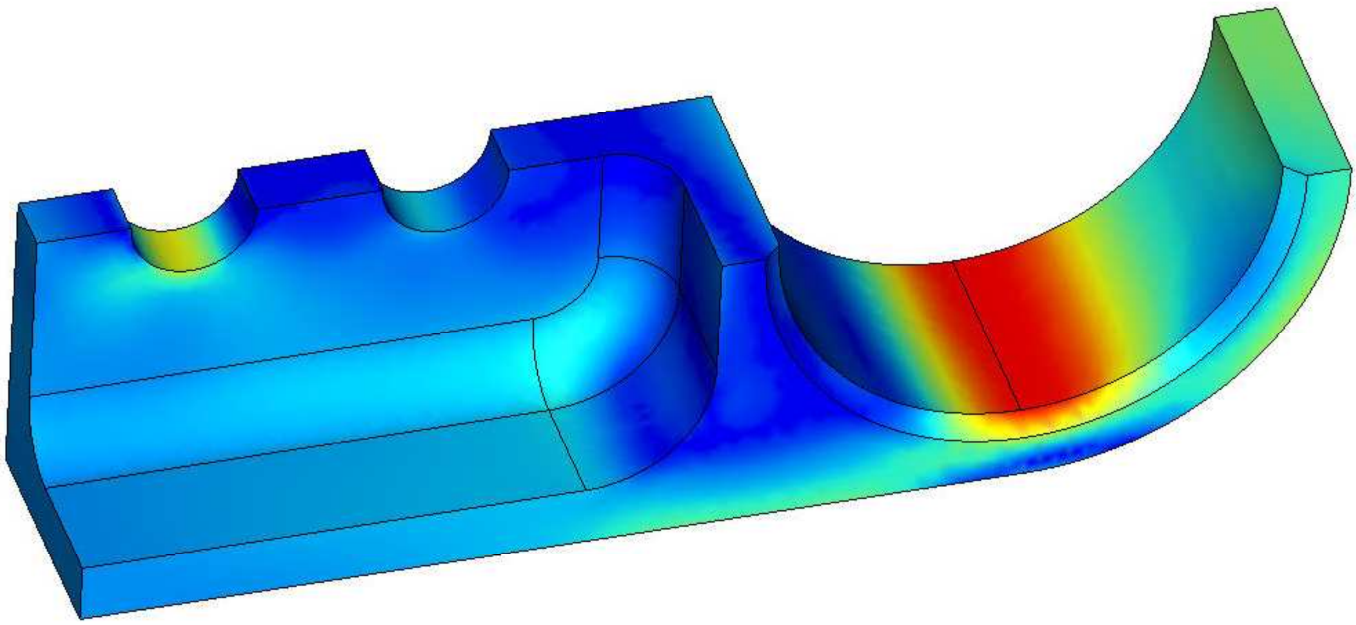
Boundary conditions applied to the one eighth model



Nodal supports shown in red, blue and green. Nodal pressure forces shown in orange.

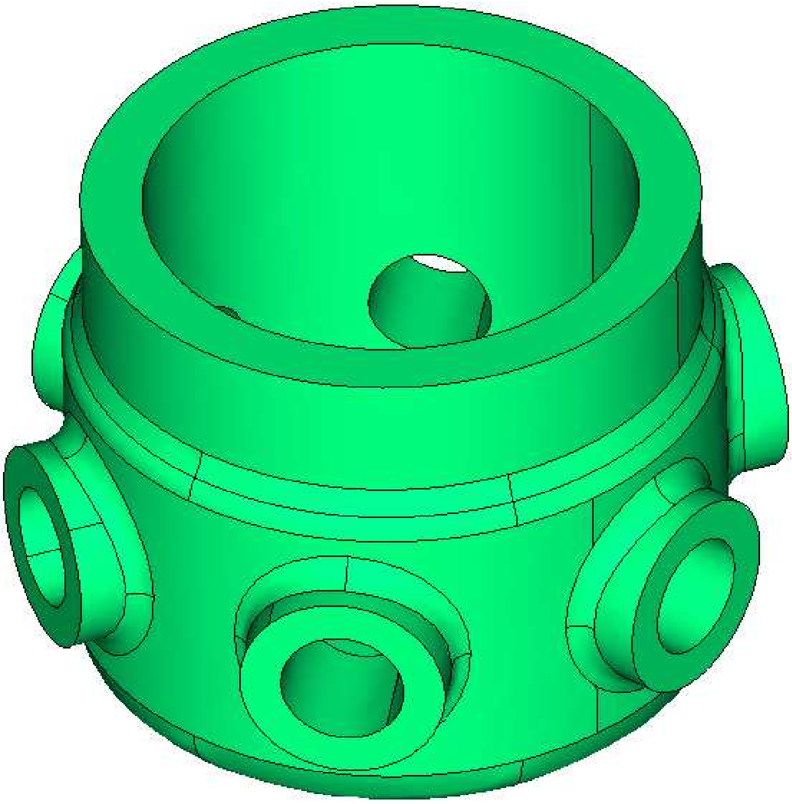
As with the 2D model with two lines of symmetry, similarly the 3D model with three planes of symmetry requires no further supports, there are no rigid body freedoms remaining in the model.

Von Mises stress contour plot

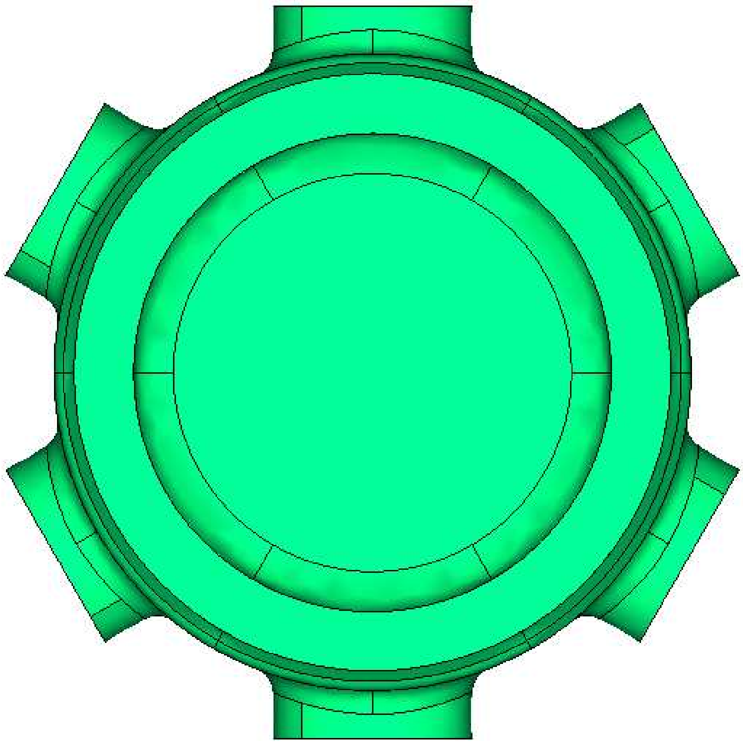


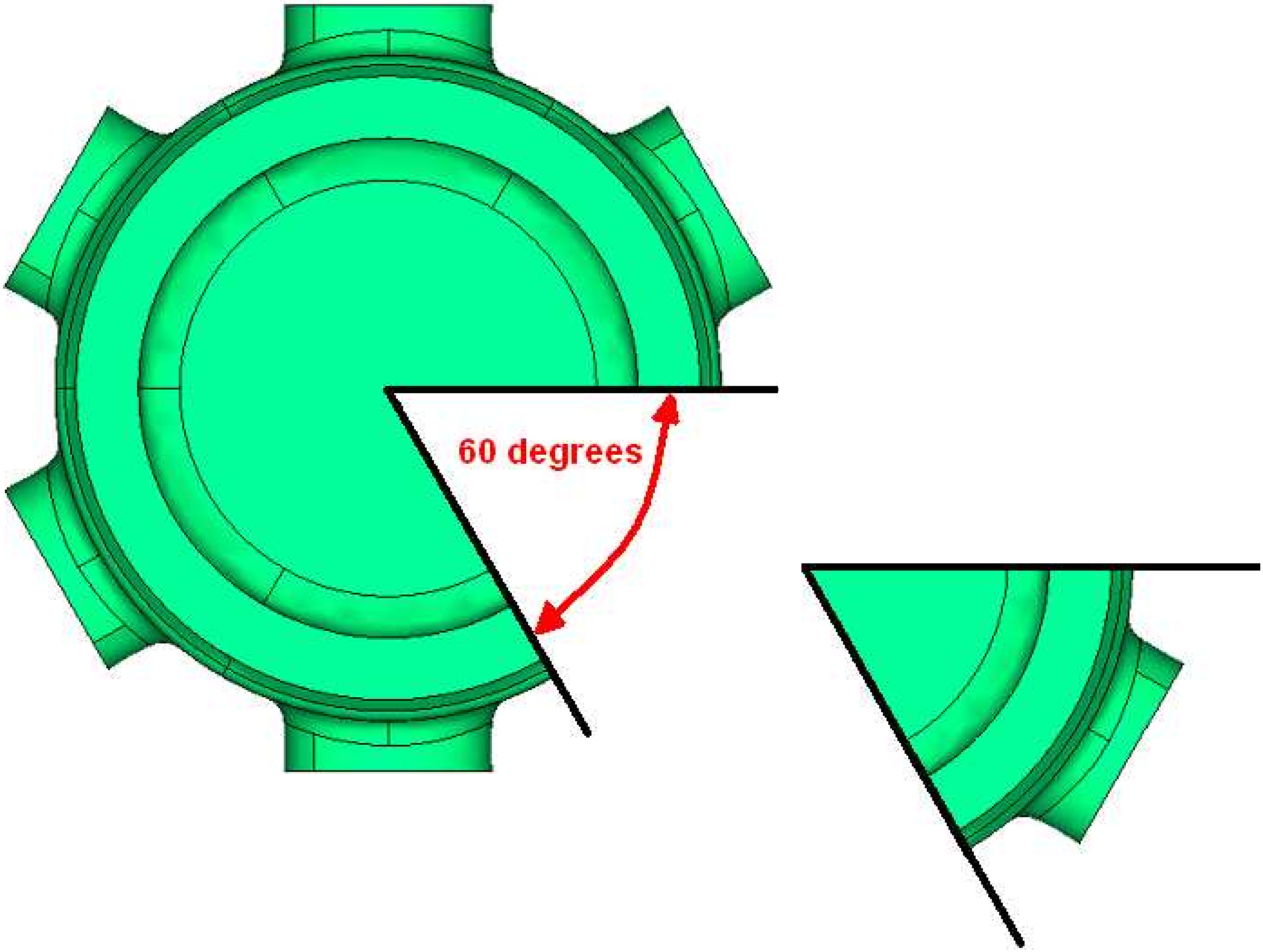
3D model with cyclic symmetry subject to tension and internal/external pressures

This kind of modelling is applicable to circular parts with repeated geometry segments around a circular axis.

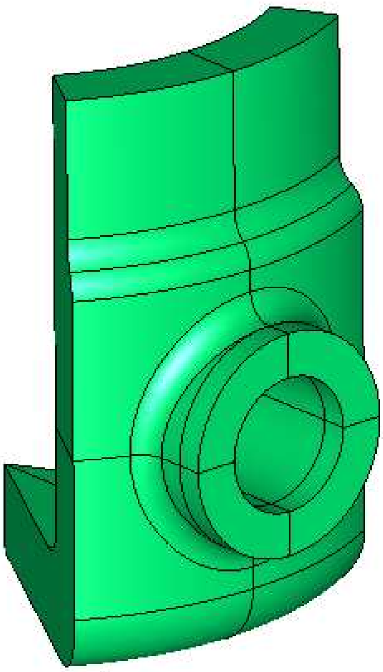


This part has geometry which repeats every 60 degrees

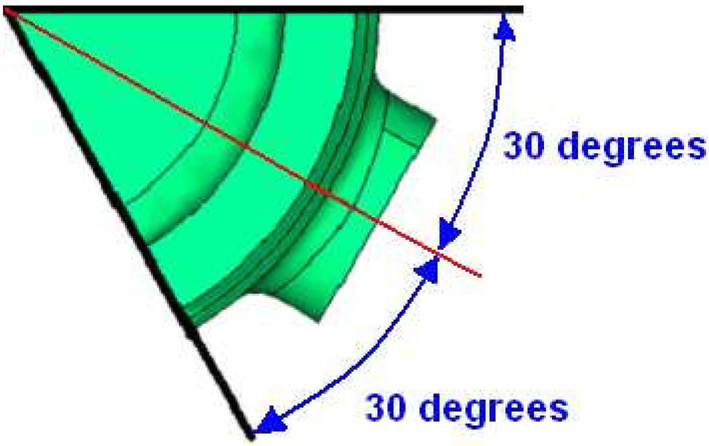


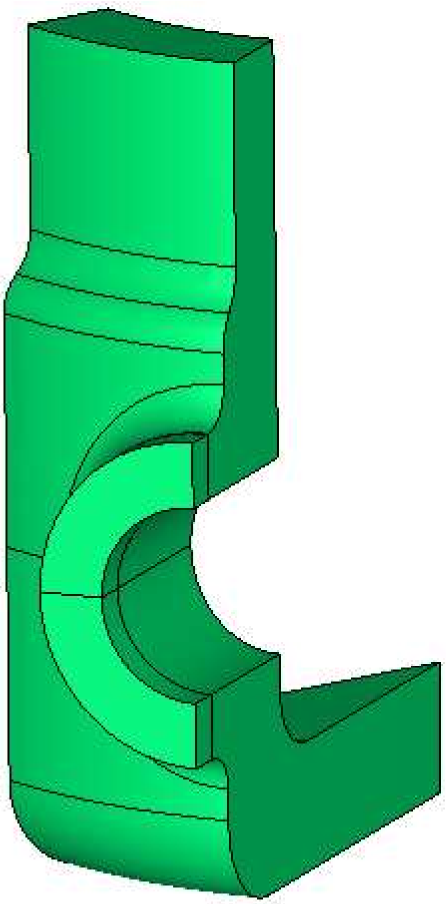


At first glance you may think that smallest model you can make and analyse is a sixty degree segment, like the one shown here.

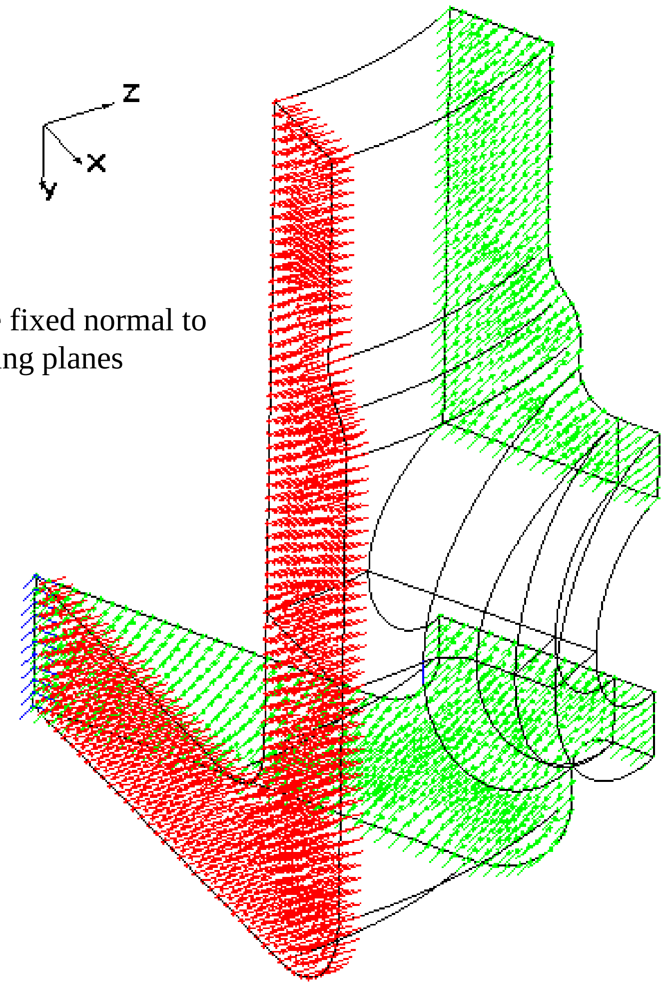


However for the simple loading case of internal to external pressure difference it is evident that results either side of the radial hole will be identical, just a mirror image of each other. Therefore the model can be reduced further still, to a thirty degree segment.





Nodal supports applied to 30 degree segment

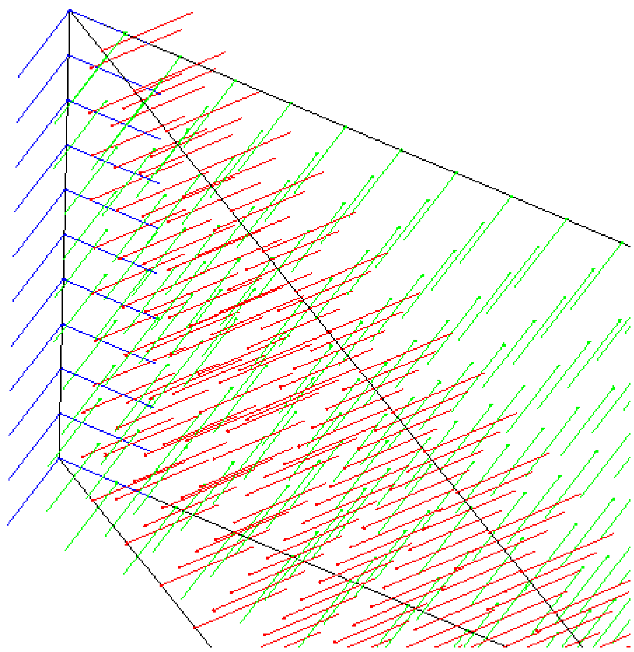


nodes are fixed normal to both cutting planes

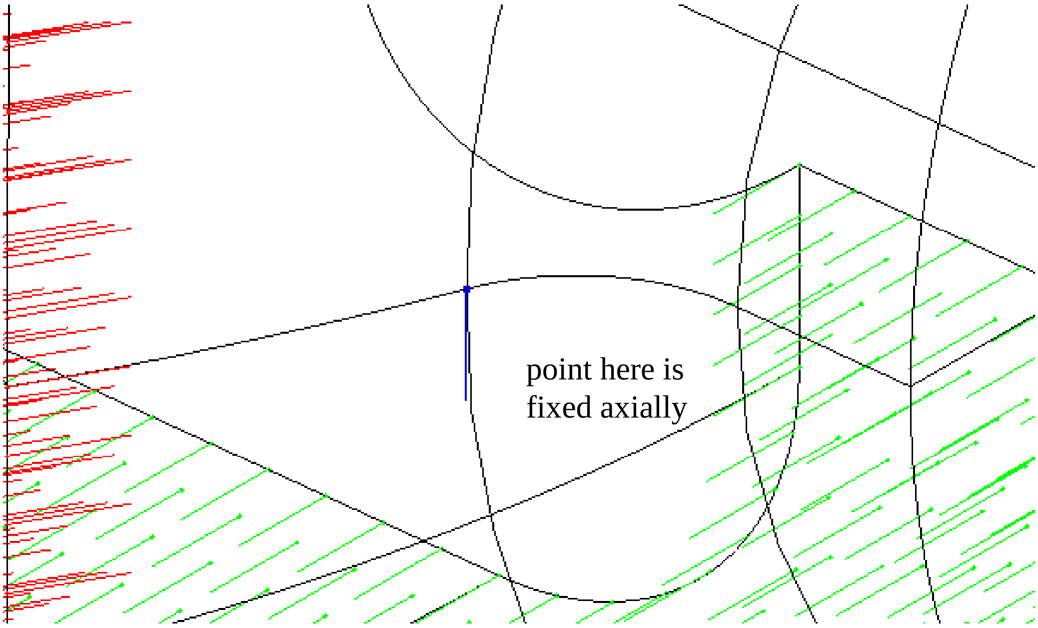
Node supports shown in red are on the XY plane and hence they are fixed in the Z direction.

Node supports shown in green are fixed normal to the cutting plane. Because this plane is not parallel to any of the global axis planes it is therefore necessary to define a local transformation and apply it to the cutting plane. The supports then act in the local direction only.

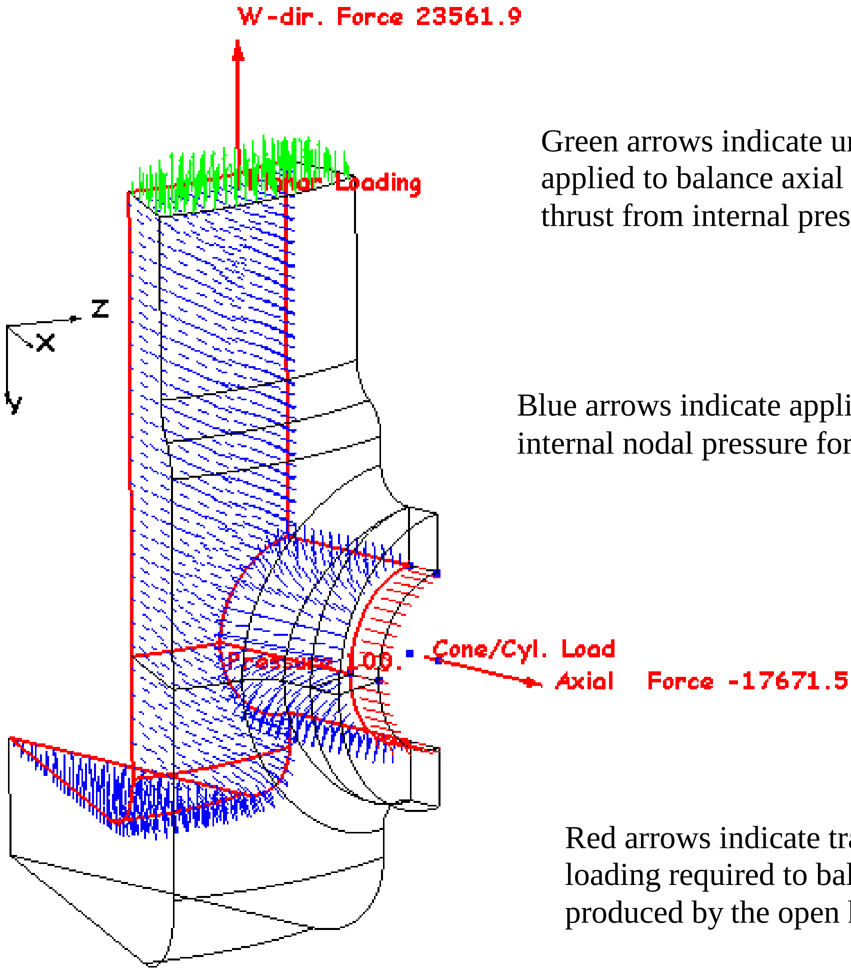
At the intersection of the two cutting planes where the radial position is zero nodes are fixed in all radial directions, only axial direction is not fixed.



With these supports applied, one rigid body freedom remains, translation along the axial direction. Therefore a single node requires to be fixed in the axial direction.



Loading Applied



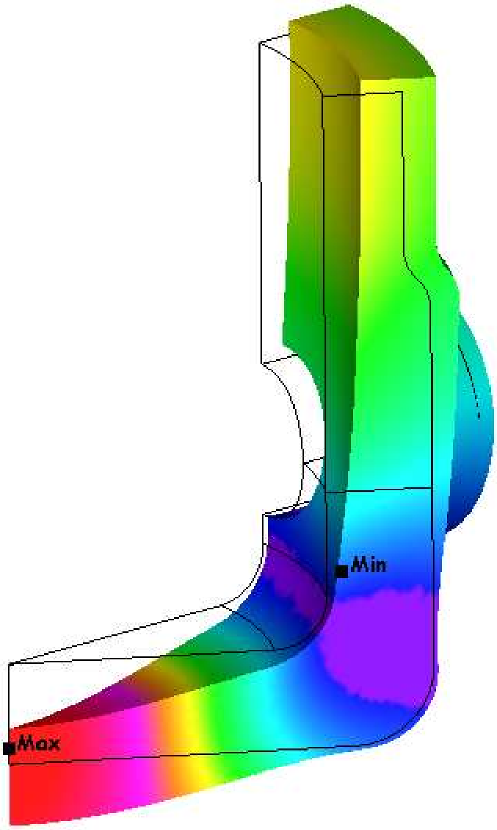
Green arrows indicate uniform pressure applied to balance axial (Y) direction thrust from internal pressure

Blue arrows indicate applied uniform internal nodal pressure forces of 100

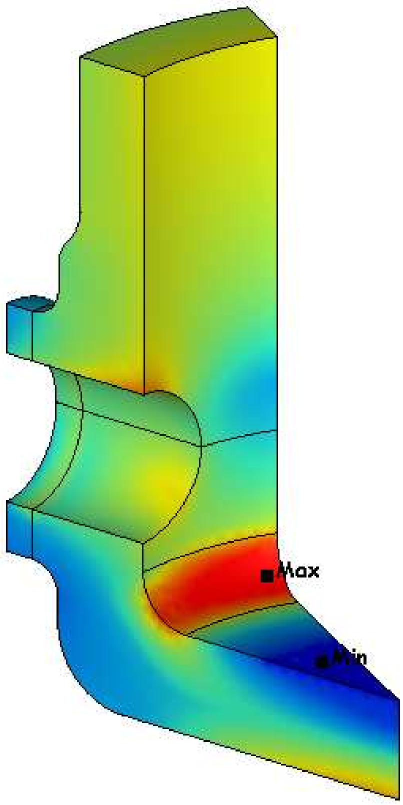
Red arrows indicate traction pressure loading required to balance thrust produced by the open hole

The model only requires a load balance in the axial (Y) direction.

Exaggerated Deformation



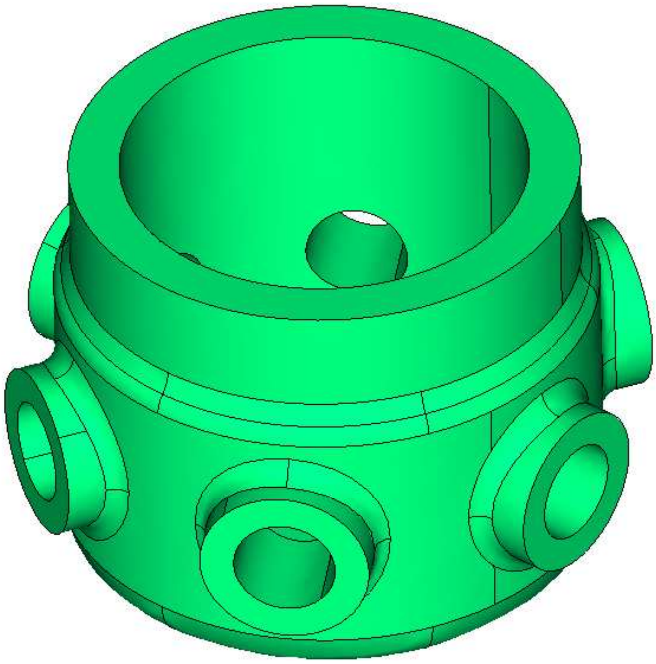
Stress Contour Result of Maximum Principal Stress

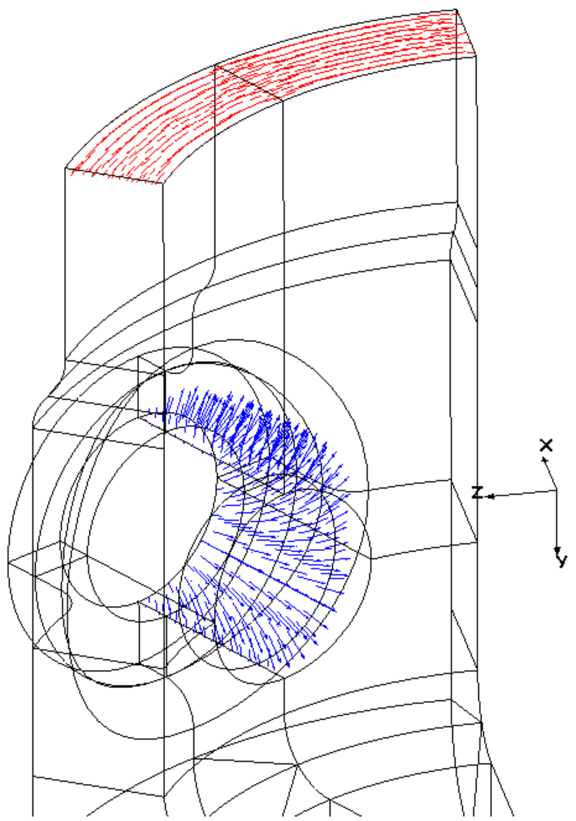


3D model with cyclic symmetry subject to tension and/or torsion

As in the previous example this kind of modelling is also applicable to circular parts with repeated geometry segments around a circular axis. Using the same basic model, repeating sixty degree segments may be used.

However unlike the previous example the sixty degree segments will no longer produce results that consist of mirrored pairs and thus a the sixty degree segment cannot be halved for analysis.





Torsional Loading on a 60 Degree Segment

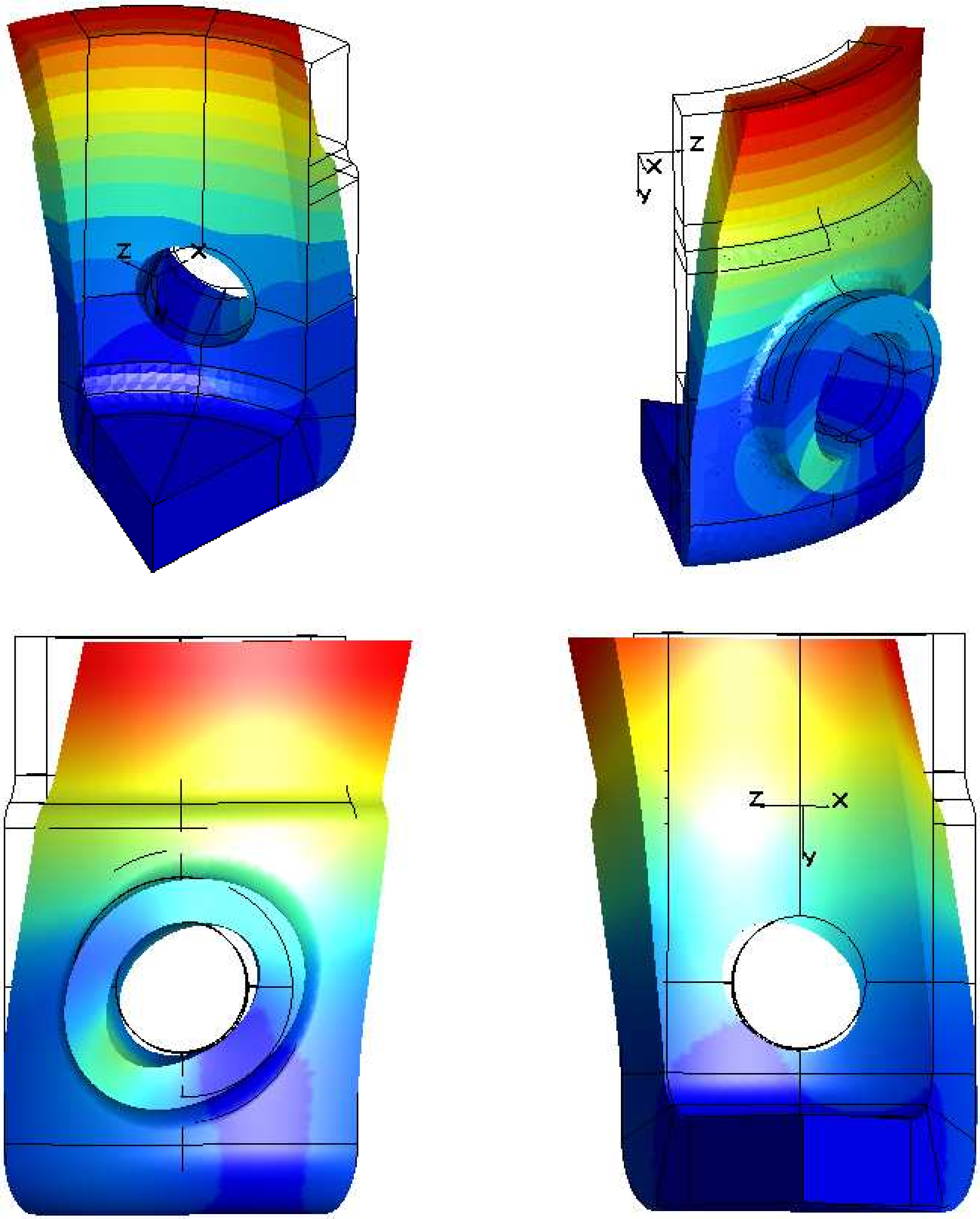
A surface traction pressure is applied on the upper annular surface, equivalent to a torsional stress on the section (shown with red arrows)

The torque that this loading applies about the central axis (Y global axis) is reacted by a lug load in one of the through holes (shown with blue arrows)

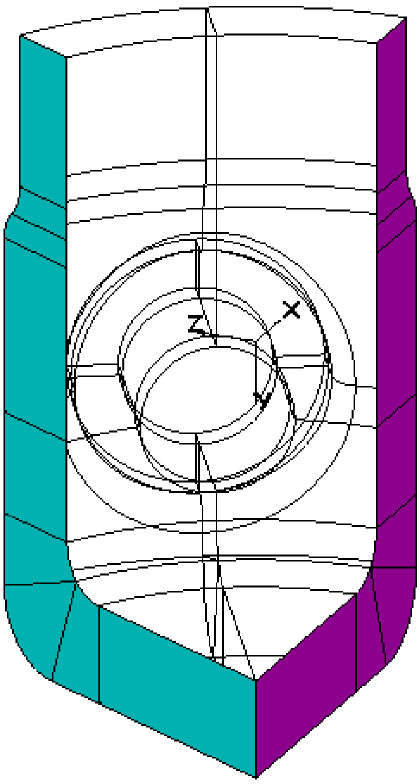
The net moment applied about the central circular axis (Y global axis) to the segment is zero

For a torque applied to the upper annulus and reacted by lug loading in the through holes, each segment will deflect as shown in the images below (exaggerated plot).

Contours of Displacement magnitude



The boundary conditions for this kind of cyclic symmetry with torsion analysis are very different to the previous example without torsion. Unfortunately unlike all previous examples in this document the procedure to achieve the desired modelling is solver dependent, even though the underlying methodology is fundamentally the same.



Consider the two cutting planes shown here (one in cyan and one in magenta), the displacement of each cutting must be linked to each other, such that points on each plane that share the same radial and axial positions have identical radial, axial and tangential deflections under loading.

This is achieved by applying constraint equations.

It is the application of these constraint equations that differs between solvers.

Where the two cutting planes intersect at zero radius and nodes are common to both planes, no constraint equations are applied.

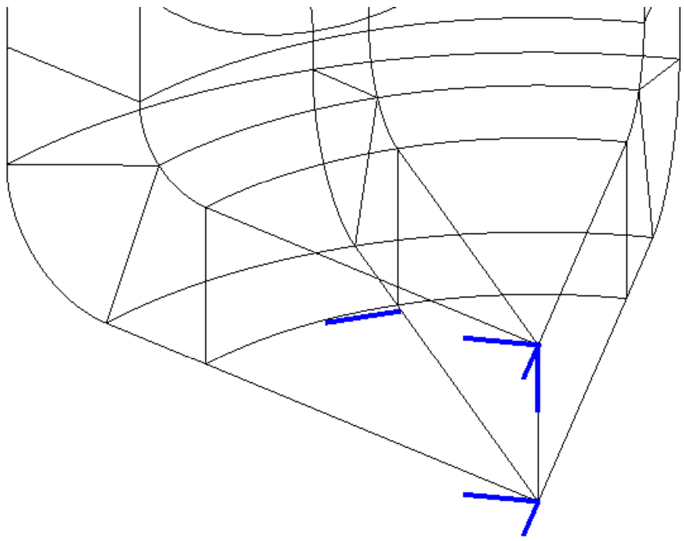
3-2-1 minimal supports can be applied to the model. Two of the three points can be selected along the intersection line with a third point not on either of the cutting planes.

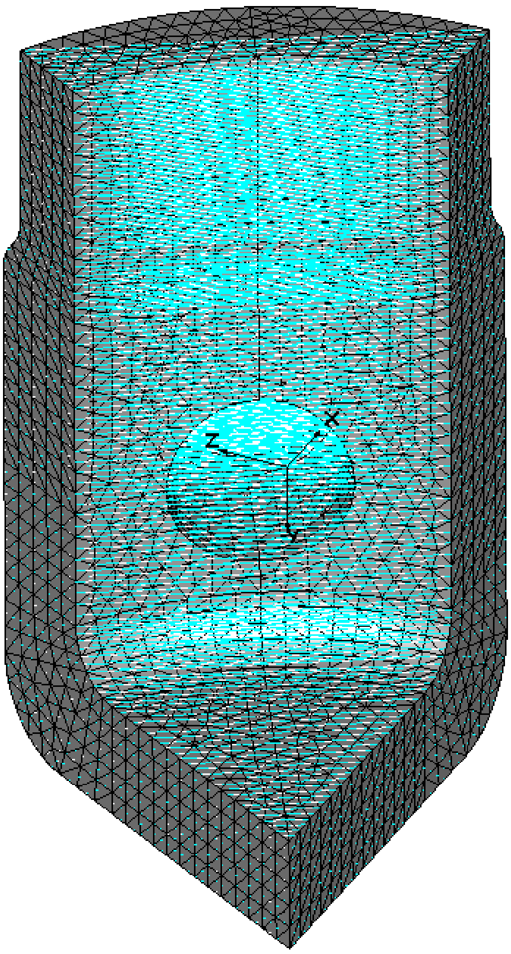
This effectively anchors the model to twist around the central model axis.

The three points selected for the minimal 3-2-1 supports are shown here. The basic principle is no different to the 3D example model discussed earlier in this document.

The first point (with 3 supports) anchors all translations whilst the second point (with 2 supports) anchors all rotation apart from twisting around the circular axis, and the third point prevents this final rigid body rotation.

In this model to get the single support direction on the third point normal to the plane formed by the three support points it was necessary to use a cartesian transformation.





For some solvers (like Lusas and Nastran) it is necessary to define a cylindrical axis transformation about the model circular axis (global Y axis in this model) and to assign this transformation to nodes on both cutting planes.

The model has been meshed such that nodes on each cutting plane match up and can be easily paired off. (Indicated by the dashed cyan lines in this image.)

Constraint equations are then defined to enforce each pair of nodes to have the same cylindrical coordinate displacements.

For other solvers (like Abaqus and CalculiX), it is not necessary to define any transformations. (the analysis will fail if you do!). Instead each cutting plane is defined as a nodal surface and assigned to a cyclic symmetry routine in which the user defines the cylindrical axis and the number of sectors that make up a complete 360 degree part.

Contour plots of Maximum Principal and Von Mises stress respectively. Note how that the contour patterns on each cutting plane are the same (as expected).